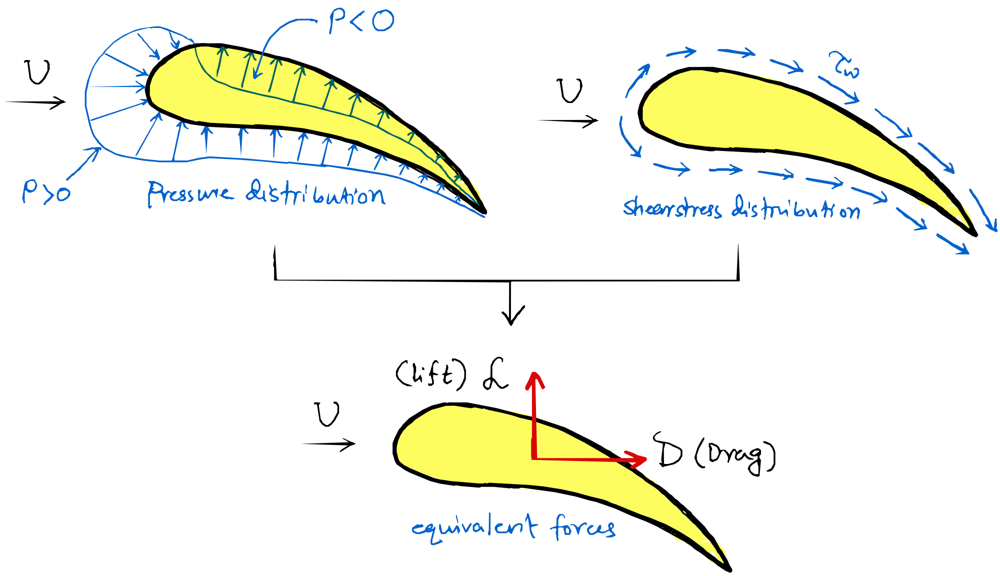
Ch-9

Lift and Drag



* Lift ($\mathcal{L}$) and drag ($\mathcal{D}$) are resultant force arising from 2 major components:
  - (a) Pressure distribution (P)
  - (b) skin friction (shear stress) distribution (τ_w)

$$\left.\begin{aligned} * \; dF_x &= (p\, dA \cos\theta) + (\tau_w\, dA \sin\theta) \\ * \; dF_y &= (-p\, dA \sin\theta) + (\tau_w\, dA \cos\theta) \end{aligned}\right\} \left(\text{Understand } \sin\theta \;\&\; \cos\theta\right)$$

$$* \; \mathcal{L} = \int dF_y = -\int p \sin\theta \, dA + \int \tau_w \cos\theta \, dA$$

$$* \; \mathcal{D} = \int dx = \int p \cos\theta \, dA + \int \tau_w \sin\theta \, dA$$

Pressure contribution   Skin friction contribution

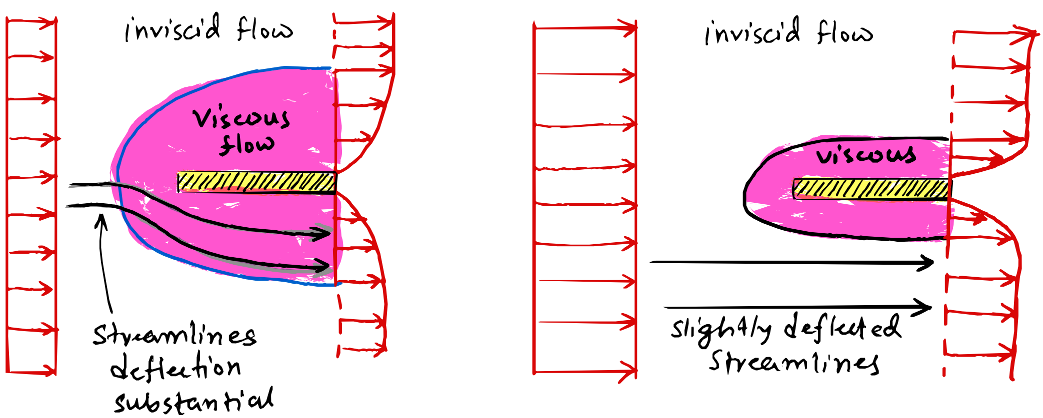
* The non-dimensional representation of these forces are

(a) lift co-efficent, $C_L = \left(\dfrac{\mathcal{L}}{\frac{1}{2}\rho U^2 A} \right)$

(b) Drag co-efficient, $C_D = \left(\dfrac{D}{\frac{1}{2}\rho U^2 A} \right)$

* General flow characteristics

  * increase in Re decreases shear (viscous) region.

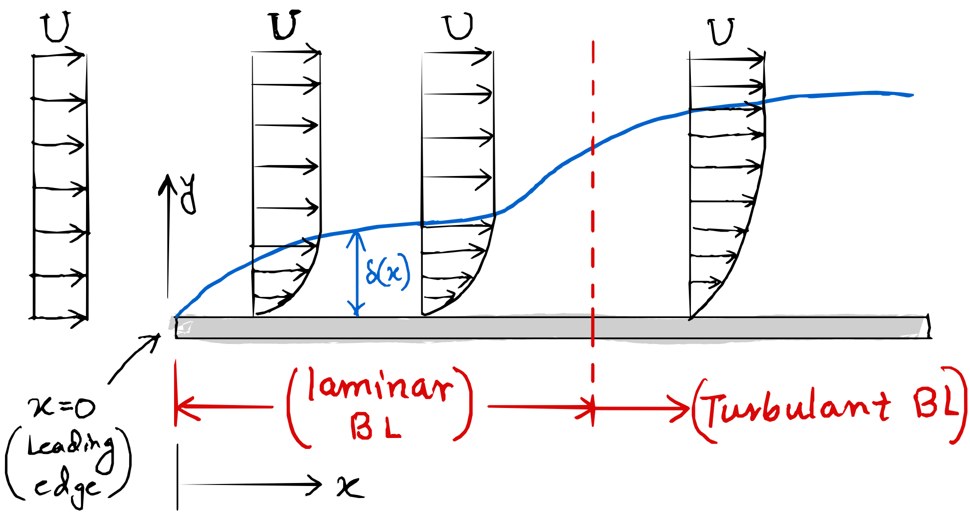


* To understand the viscous shear stress we need to understand how boundary layer forms and its geometric details.

↓

(Thickness of the boundary layer)

Flow over flat plate



* The flow is 2-D, need x and y component of N-S to know the velocity field.

Elliptic PDEs

$$\begin{cases} x: \; u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} = -\dfrac{1}{\rho}\left(\dfrac{\partial P}{\partial x}\right) + \left(\dfrac{\mu}{\rho}\right)\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}\right) \\[2ex] y: \; u\dfrac{\partial v}{\partial x} + v\dfrac{\partial v}{\partial y} = -\dfrac{1}{\rho}\left(\dfrac{\partial P}{\partial y}\right) + \left(\dfrac{\mu}{\rho}\right)\left(\dfrac{\partial^2 v}{\partial x^2} + \dfrac{\partial^2 v}{\partial y^2}\right) \end{cases}$$

* Also we need continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

* Assumptions
  1. steady
  2. incompressible
  3. No gravity force

* what are the boundary conditions

$$u = v = 0 \quad (y = 0)$$

$$u \to U \quad (y \to \infty)$$

* Prandtl simplified the equations and his student Blasius (1908) was able to solve the problem for laminar boundary layer.

Prandtl's simplification to boundary layer problem

* Assumed : The boundary layer is very thin

$$(Re \gg 1)\left\{\begin{array}{l} \to \text{Thus } v \ll u \\ \to \text{and } \frac{\partial}{\partial x} \ll \frac{\partial}{\partial y} \end{array}\right\} \text{Think !!}$$

* Imposing these assumptions the N-S eqn reduces to

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \quad \left(\text{Pressure is almost constant}\right)$$

(praraboolic PDE)

* By moving from elliptic to parabolic PDE we loose the systems sensitivity to affect up stream.

* The velocity profile at any location-x should be similar, $\left(\frac{u}{U}\right) = \phi\left(\frac{y}{\delta}\right)$

Blasius's solution to Boundary layer problem

* Blasius showed that the velocity field $\frac{u}{U}$ can be expressed as a function of single variable $y\left(\sqrt{\frac{U}{\nu x}}\right)$, known as similarity variable.

  → This claims that the boundary layer thickness δ scales with $\sqrt{x}$ and $1/\sqrt{U}$.

$$\delta \sim \sqrt{\frac{\nu x}{U}}$$

→ This is an ansatz (assumption)

* Assuming $\eta = y\left(\sqrt{\frac{U}{\nu x}}\right)$ the velocity field can be expressed as,

$$\frac{u}{U} = f'(\eta) \qquad \left[\text{Here, } f' = \frac{\partial f}{\partial \eta}\right]$$

$$\Rightarrow u = U f'(\eta)$$

* Since u is known we can obtain a streamfunction as,

$$\psi = \int u \, dy = U \int f'(\eta) dy$$

Now, $\frac{d\eta}{dy} = \left(\sqrt{\frac{U}{\nu x}}\right) \quad \therefore \ dy = \left(\sqrt{\frac{\nu x}{U}}\right) d\eta$

$$\Rightarrow \psi = U \cdot \int f'(\eta) \sqrt{\frac{\nu x}{U}} \, d\eta$$

$$\Rightarrow \psi = U\sqrt{\frac{\nu x}{U}}\, f(\eta) = \sqrt{\nu x U}\, f(\eta)$$

* From stream function we can obtain v as

$$v = -\frac{\partial \psi}{\partial x} = -\left[\sqrt{\nu x U}\,\frac{df}{dx} + f\,\frac{d}{dx}\sqrt{\nu x U}\right]$$

$$= -\left(\sqrt{\nu x U}\right)\frac{df}{d\eta}\frac{d\eta}{dx} - f\cdot\sqrt{\nu U}\cdot\frac{1}{2}\frac{1}{\sqrt{x}}$$

$$= -\sqrt{\nu x U}\cdot f'\, y\sqrt{\frac{U}{\nu}}\left(-\frac{1}{2}\right)\frac{1}{x^{3/2}} - \frac{1}{2}f\sqrt{\frac{\nu U}{x}}$$

$$= \sqrt{\frac{\nu U}{4x}}\left(\eta f' - f\right)$$

* With known expressions for u and v the boundary layer problem can be shown to be reduced as

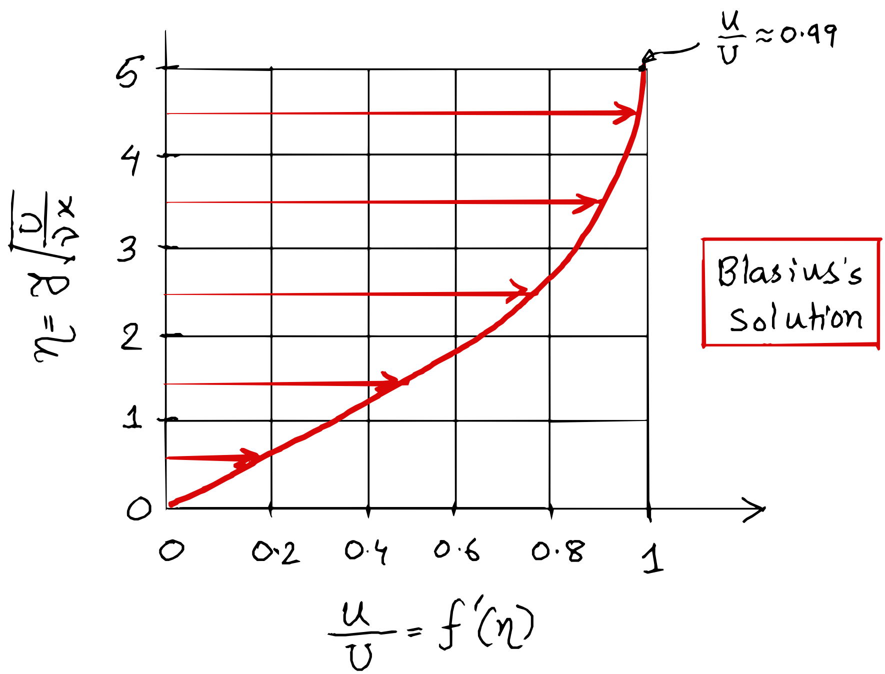
$$2f''' + ff'' = 0 \quad \rightarrow \quad \text{1 homogeneous non-linear PDE}$$

where,

$$\left.\begin{aligned} f(\eta=0) &= 0 \\ f'(\eta=0) &= 0 \\ f'(\eta\to\infty) &\to 1 \end{aligned}\right\} \text{3 boundary cond}^{\text{s}}$$

* Though analytical solution of above equation (PDE & BCs) is not possible, numerical solution can be obtained very easily.

* The numerically obtained solution is shown in figure below.



* The results shown are non-dimensional and is a single plot. But it gives the velocity profile at different x-values.

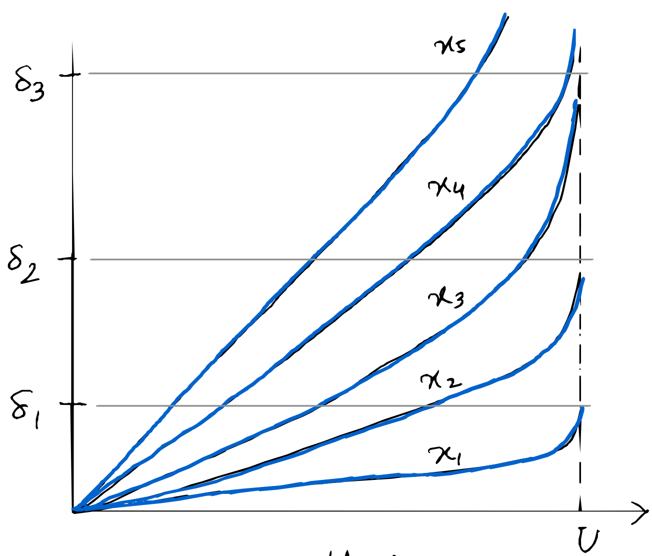
* The boundary layer thickness shows that

$$\delta = (5)\sqrt{\frac{\nu x}{U}} \Rightarrow \left(\frac{\delta}{x}\right) = \left(\frac{5}{\sqrt{Re_x}}\right)$$

Comments on the Blasius's solution

* Even though the solution (Graph) shows single plot it is presented as a non-dimensional general solution.

  → It can produce specific velocity profile at any given x value (and U).



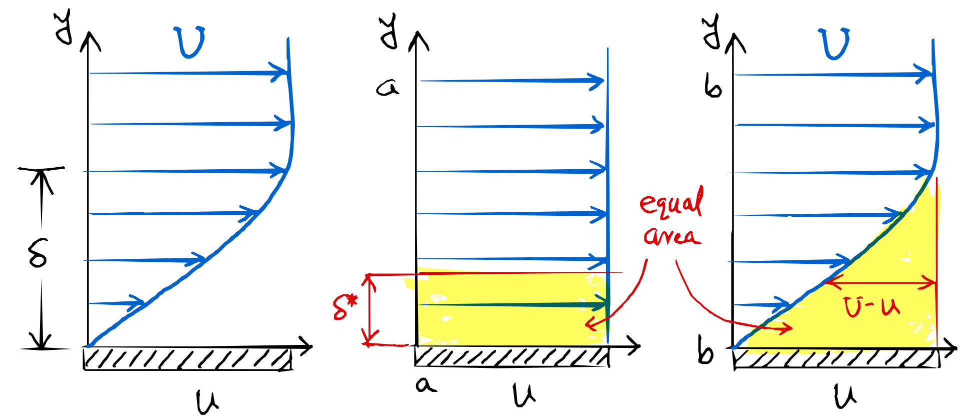
* From known velocity profile the shear stress can be calculated easily as

$$\tau_w = 0.332\, U^{3/2}\left(\sqrt{\frac{\rho\mu}{x}}\right)$$

* Why define boundary layer thickness @ $u/U \approx 0.99$.

There is no answer to this question. An appropriate way to deal with boundary layer thickness is "boundary layer displacement thickness δ^*"

→ It is determined directly from mass balance.



* Due to deficit in flow velocity near solid surface the fluid mass flowing through bb is less than aa (for uniform flow). Thus if we move the solid by a displacement δ^* then rest of the fluid can be treated as inviscid.

$$U b \delta^* = \int_0^\infty (U-u)\, b\, dy$$

$$\Rightarrow \quad \delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy$$

* We can apply same idea for momentum flow.
→ The displacement thickness is called "boundary layer momentum thickness Θ"

* Balancing the momentum gives

$$\rho b U^2 \Theta = \rho b \int_0^\infty u(U-u)\,dy$$

$$\Rightarrow \Theta = \int_0^\infty \frac{u}{U}\left(1-\frac{u}{U}\right) dy$$

Blasius solution for δ^* and Θ

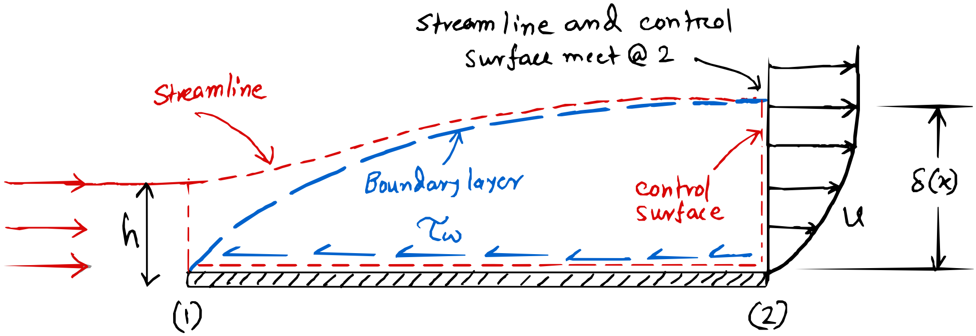
* From the velocity profile known (Blasius's solution) it is straight forward show that the displacement thickness and momentum thickness can be expressed as

$$\frac{\delta^*}{x} = \frac{1.721}{\sqrt{Re_x}}$$

$$\frac{\Theta}{x} = \frac{0.664}{\sqrt{Re_x}}$$

Momentum Integral Boundary layer Equation for flow over flat plate

* Exact solution is hard (or impossible).
* Need "alternative approximate method"



* The drag force due to skin friction

$$\sum F_x = -D = -\int_{Plate} \tau_w \, dA = -b \int_{Plate} \tau_w \, dx$$

* x-momentum:

$$-D = \rho \int_{(1)} U(-U)\, dA + \rho \int_{(2)} u^2 \, dA$$

$$\Rightarrow \quad D = \rho U^2 b h - \rho b \int_0^{\delta} u^2 \, dy$$

(Unknown) How to get it?

* Conservation of mass, $Uh = \int_0^\delta u\,dy$

* Substitution of above expression gives

$$\mathcal{D} = \rho U b \int_0^\delta u\,dy - \rho b \int u^2\,dy = \rho b \int_0^\delta u(U-u)\,dy$$

* Again we know,

$$\Theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \Rightarrow U^2\Theta = \int_0^\delta u(U-u)\,dy$$

* Thus, $\mathcal{D} = \rho b U^2 \Theta$ (Simple, approximate)

$\longrightarrow$ It is valid for both laminar and turbulant boundary layer.

* Since Θ is not a constant, the above expression gives the local drag force, which can be used to obtain the x-variation of shear stress. Differentiating the above expression with respect to x we get,

$$\frac{d\mathcal{D}}{dx} = (\rho b U^2)\frac{d\Theta}{dx}$$

* Now from $-\mathcal{D} = -\int \tau_w\,dA = -b\int \tau_w\,dx$ we can say, $d\mathcal{D} = b\,\tau_w\,dx$

* Substitution gives, $b\tau_w = \rho b U^2 \left(\frac{d\Theta}{dx}\right)$ or

$$\tau_w = \rho U^2 \left(\frac{d\theta}{dx}\right)$$

* If velocity profile is known then the below equations can be used to obtain the shear stress and drag force:

$$\tau_w = \rho U^2 \left(\frac{d\theta}{dx}\right) \qquad \Theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy.$$

$$\mathcal{D} = \rho b U^2 \Theta$$

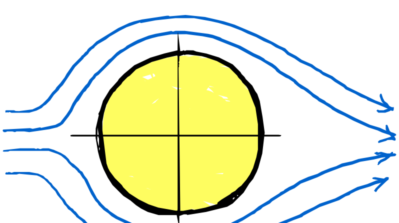
* Fortunately a crude estimation of velocity profile gives a reasonable value for $\mathcal{D}$ and τ_w (see example 9.4).

* Even though a rough estimation is possible for τ_w, what about pressure distribution (pressure contribution).

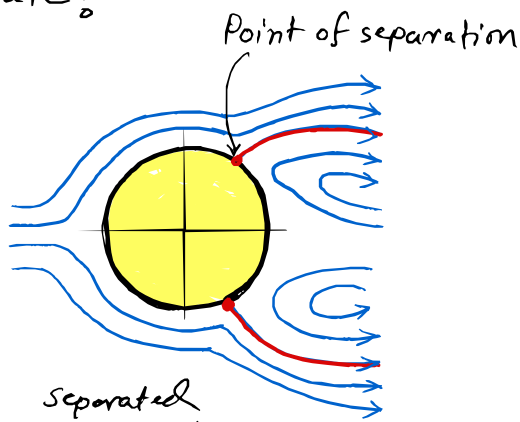
* For most applications pressure contribution is smalar and the combined effect of pressure and shear stress need to be determined experimentally.

* The momentum integral approach is simple and capable of resonable approximation of the resultant forces (lift and drag) due to skin friction, it can not be applied when "boundary layer seperates".

* Why boundary layer seperate?



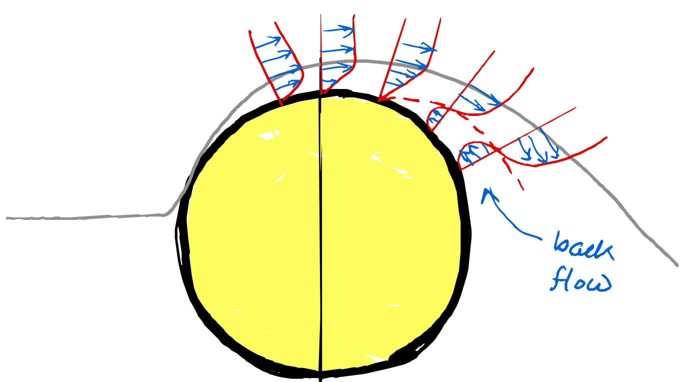
attached boundary layer



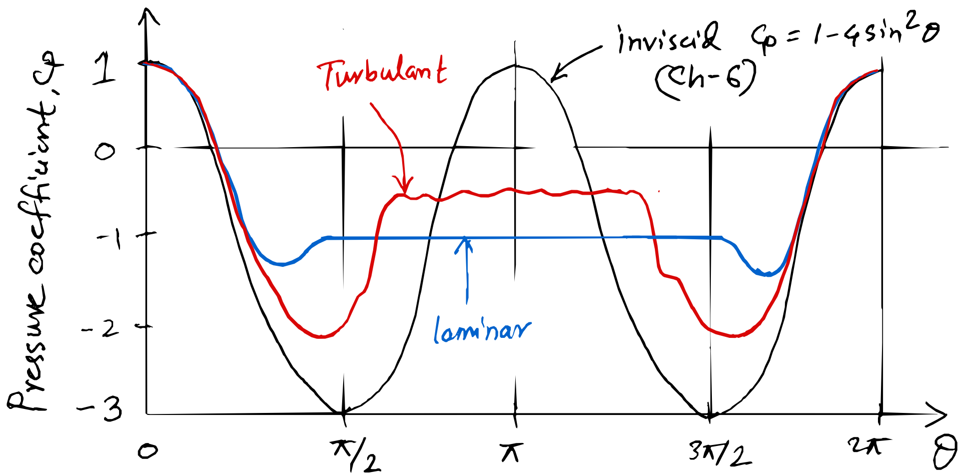
seperated boundary layer.

→ adverse pressure gradient.

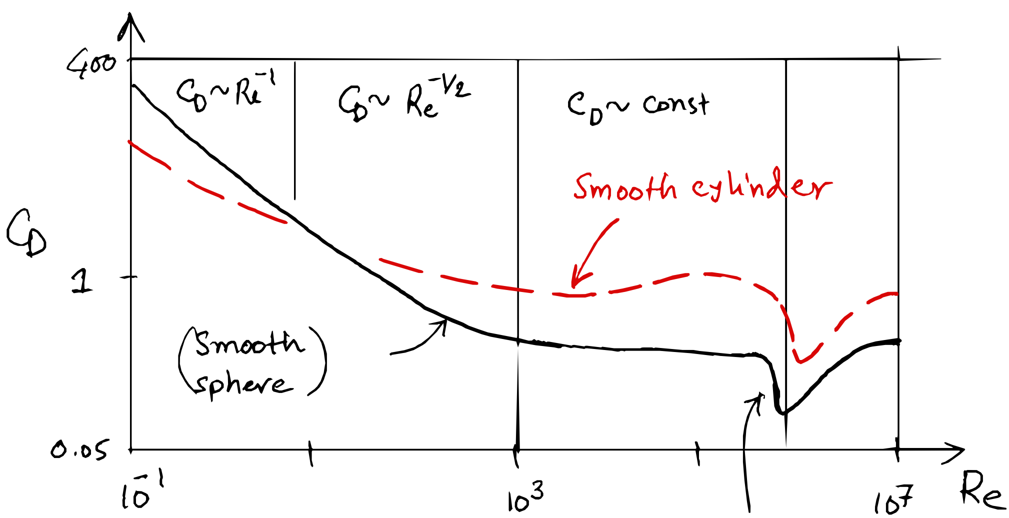
(what will happen if a fluid is not allowed to increase speed, while a force is applied)



* Due to the separation of the viscous boundary layer the pressure near the back goes down.



* Usually C_L and C_D are experimentally obtained

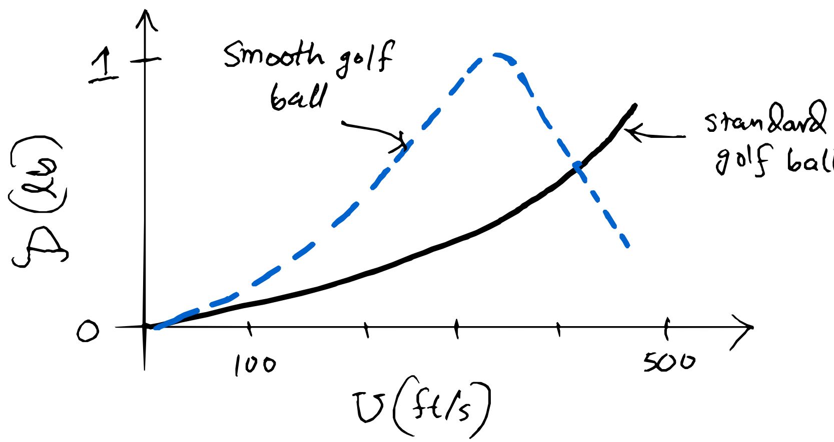


Turbulance causes delay in separation of the boundary layer.

* What other way we can make the boundary layer turbulant?

→ Roughness !!

→ Why golf balls are not smooth?



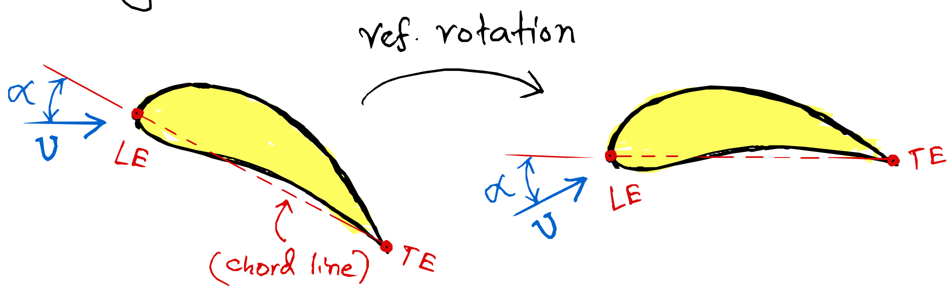
* For more on effect of solid shape, roughness and size see figures/table from textbook.

(Presented in supporting slides)

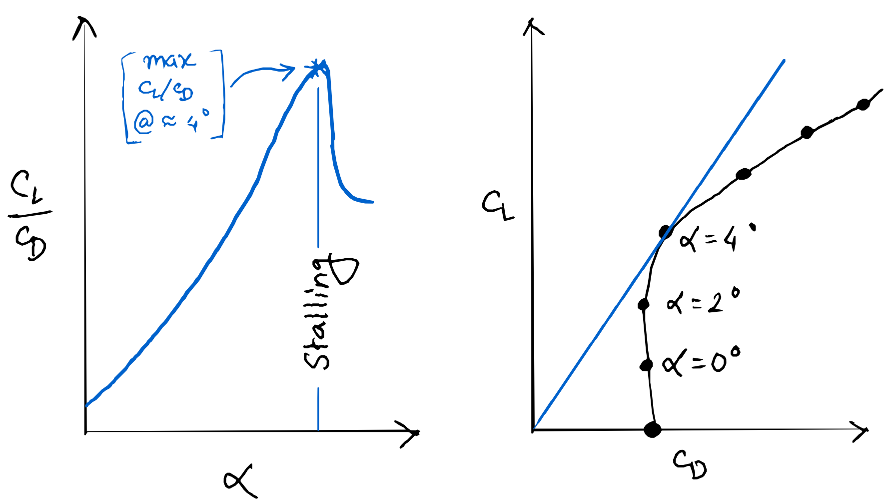
* what about lift?

→ Idea is same. Do experiment and apply model/prototype transformation.

* Design criteria is based on the lift to drag ratio.



* Example of variation of C_L/C_D variation with angle of attack (α) is shown below.

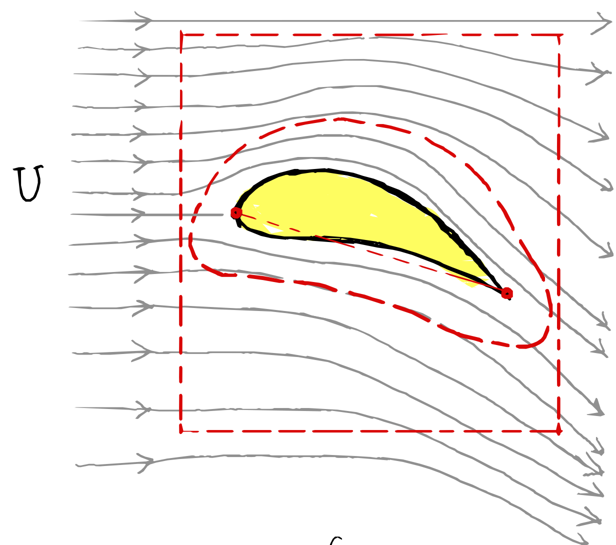


* Remember kutta-Jokowski theorem?

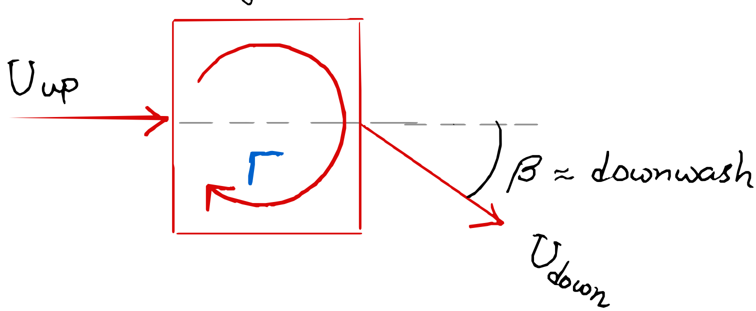
$$\mathcal{L} = \rho U \Gamma$$

Circulation

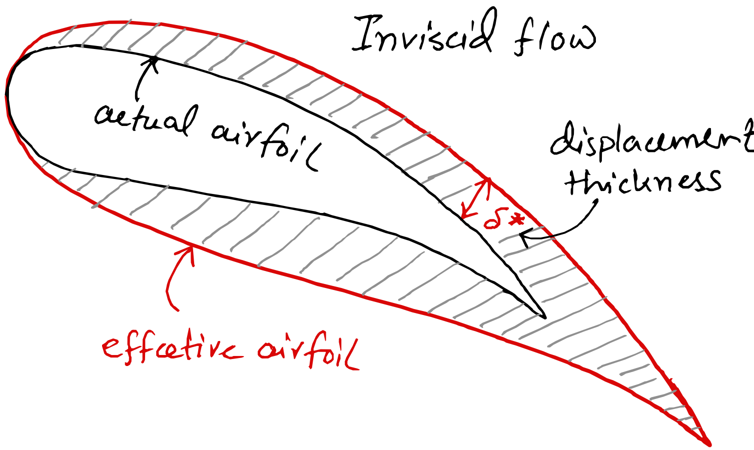
* Often Circulation is represented as downwash (Deflection of upstream fluid flow)



$$\Gamma = \oint \vec{v} \cdot d\vec{s}$$



* Effective airfoil shape



* Thin airfoil theory (vortex sheet along mean camber line)