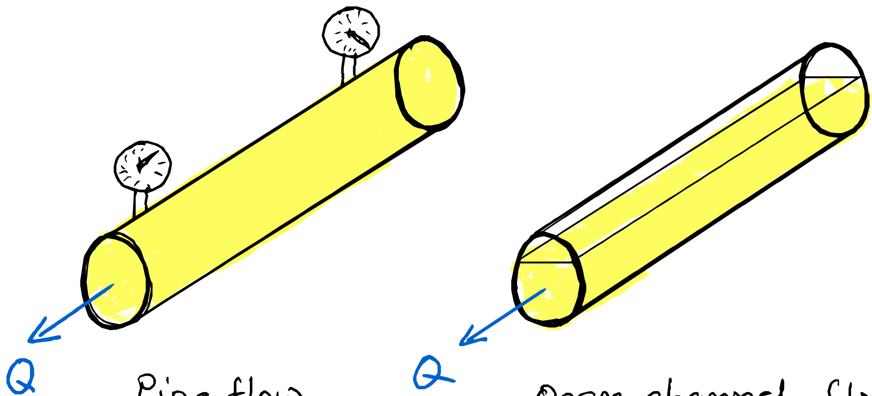
Ch-8

* Pipe flow Vs open channel flow

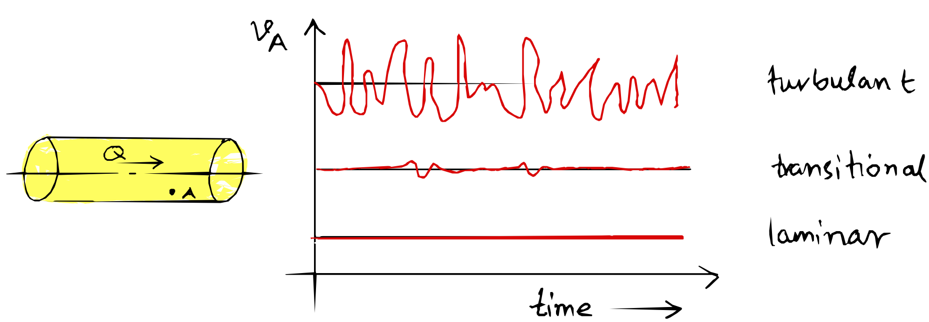


Pipe flow
(Pressure driven)

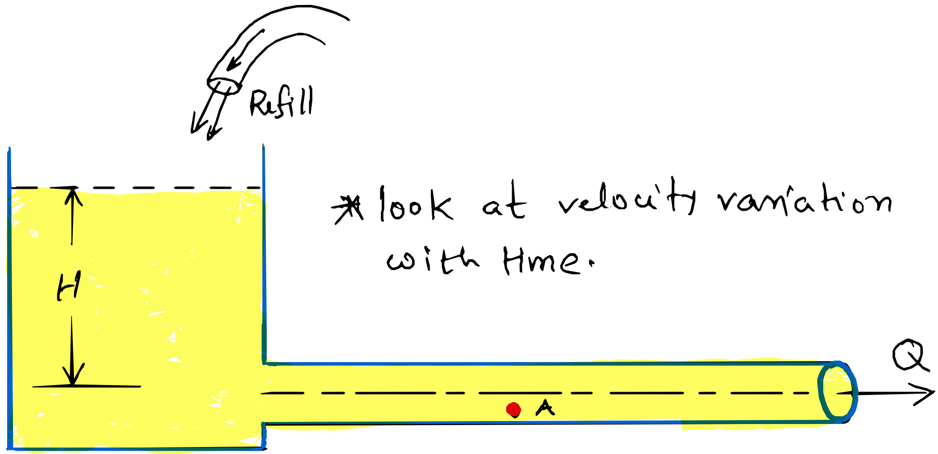
Open channel flow
(Gravity driven)

* Pipe flow characteristics

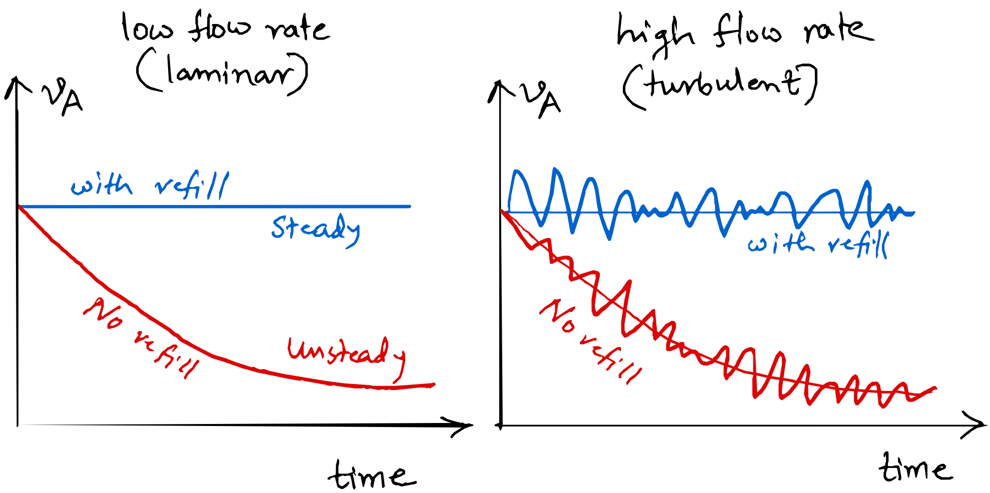
→ Laminar and turbulant flow
→ Entrance region and fully developed flow
→ steady and unsteady flow



* Unsteady Vs turbulant flow



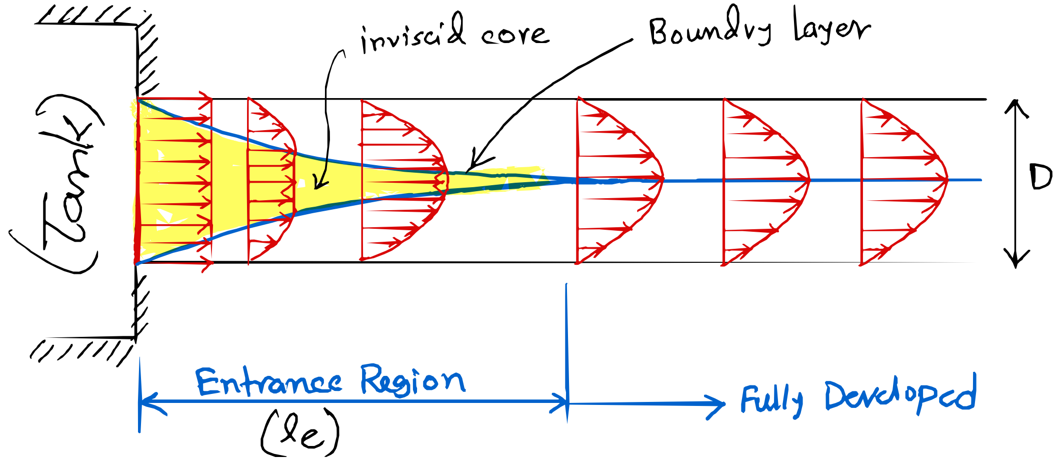
* look at velocity variation with time.

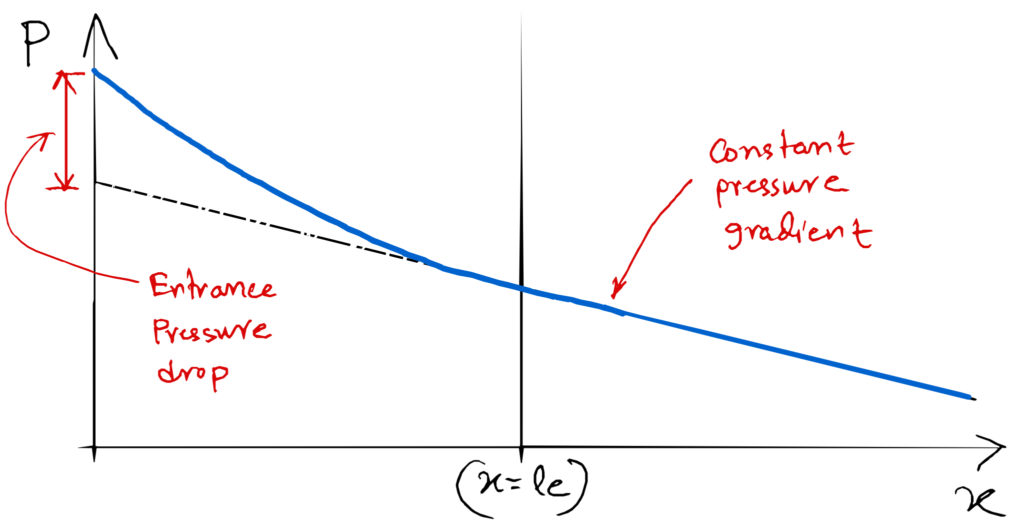


* An unsteady flow can be laminar or turbulant depending on the flow rate.

* For pipe flow $\begin{cases} 0 \leq Re \leq 2100 \rightarrow \text{(laminar)} \\ 4000 \leq Re \longrightarrow \text{(Turbulant)} \end{cases}$

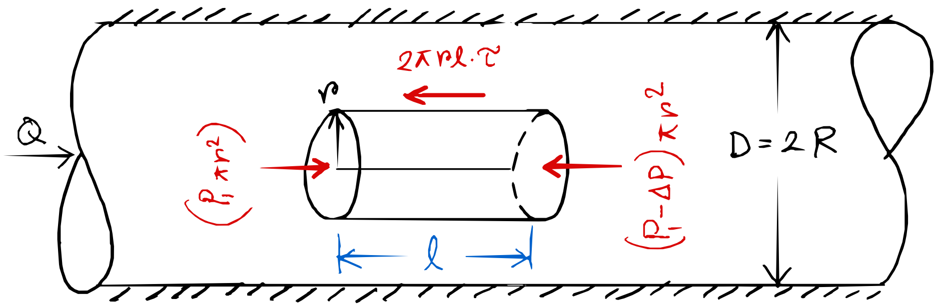
* Entrance region and fully developed flow (Developing region)





For $\begin{cases} \text{laminar flow,} & \left(\dfrac{l_e}{D}\right) = 0.06\,(Re) \\ \text{turbulent flow,} & \left(\dfrac{l_e}{D}\right) = 4.4\,(Re)^{1/6} \end{cases}$

* Most simplest form is laminar- fully developed flow:



* From Force balance:

$$P_1 \pi r^2 - P_1 \pi r^2 + \Delta P \pi r^2 - 2\pi r l \tau = 0$$

$$\Rightarrow \left(\frac{\Delta P}{l}\right) = \left(\frac{2\tau}{r}\right)$$

→ For fully developed flow $\left(\frac{\Delta P}{l}\right)$ is constant

* Thus, $\frac{2\tau}{r} = c \Rightarrow \tau = \left(\frac{cr}{2}\right)$

* How to obtain c ?? what is known about τ ?

* (a) $\tau(r=0) = 0$ (Gives nothing)

(b) $\tau(r=R) = \tau_w$, $\tau_w = \frac{cR}{2} \Rightarrow c = \left(\frac{2\tau_w}{R}\right)$

* Thus, $\tau = \tau_w \cdot \left(\frac{2}{R}\,\frac{r}{2}\right) = (\tau_w)\frac{r}{R}$

* Then $\frac{\Delta P}{l} = \left(\frac{2\tau}{r}\right) = \left(\frac{2\tau_w r}{r \cdot R}\right) = \left(\frac{2\tau_w}{R}\right)$

$$\Rightarrow \Delta P = \left(\frac{2 l \tau_w}{R}\right) \longrightarrow \text{Pressure drop and shear stress relation}$$

* Now we know, $\tau = -\mu \frac{\partial u}{\partial r}$

$$\Rightarrow \left(\frac{\partial u}{\partial r}\right) = -\left(\frac{\tau}{\mu}\right) = -\left(\frac{\tau_w r}{\mu R}\right)$$

$$\Rightarrow \left(\frac{\partial u}{\partial r}\right) = -\frac{1}{2\mu}\left(\frac{\Delta P}{l}\right) r \qquad \left[\text{Since, } \Delta P = \frac{4 l \tau_w}{D}\right]$$

* Integration gives,

$$u = -\frac{1}{4\mu}\left(\frac{\Delta P}{l}\right) r^2 + C_1$$

* Now $r = R$, $u = 0$ $\Rightarrow 0 = -\frac{1}{4\mu}\left(\frac{\Delta P}{l}\right) R^2 + C_1$

$$\Rightarrow C_1 = \frac{1}{4\mu}\left(\frac{\Delta P}{l}\right) R^2$$

* Velocity Profile, $u = -\frac{1}{4\mu}\left(\frac{\Delta P}{l}\right) r^2 + \frac{1}{4\mu}\left(\frac{\Delta P}{l}\right) R^2$

$$\Rightarrow u = \frac{R^2}{4\mu}\underbrace{\left(\frac{\Delta P}{l}\right)}_{= -\left(\frac{\partial P}{\partial z}\right)}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

$$\Rightarrow u = \frac{R^2}{4\mu}\left(-\frac{\partial P}{\partial z}\right)\left[1 - \left(\frac{r}{R}\right)^2\right]$$

Same as derived in Ch-6 !!
(From N-S equation)

* whow to get flow rate and average velocity?

$$Q = \int \vec{u} \cdot d\vec{A} = \left(\frac{\pi R^4}{8\mu}\right)\left(\frac{\Delta P}{l}\right) = \left(\frac{\pi D^4 \, \Delta P}{128 \mu l}\right)$$

and, $$v_{avg} = \frac{1}{A}\int \vec{u} \cdot d\vec{A} = \left(\frac{D^2 \Delta P}{32 \mu l}\right)$$

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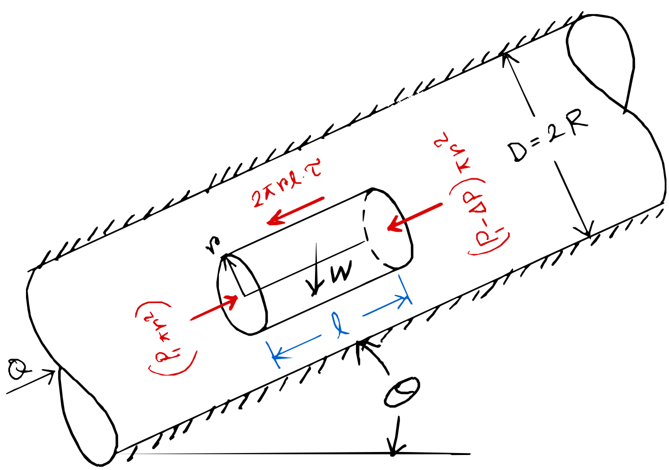
* what will change if the tube in inclined?



* Force balance: $(P_1 \pi r^2) - (P_1 - \Delta P)\pi r^2 - 2\pi r l \, \tau - \rho g \sin\theta = 0$

$\Rightarrow \Delta p \, \pi r^2 - \rho g \sin\theta = 2\pi r l \, \tau$

$\Rightarrow \left(\frac{\Delta P - \rho g l \sin\theta}{l}\right) = \left(\frac{2\tau}{r}\right)$

(Pressure term adjustment due to elavation.)

* Thus, $v_{avg} = \dfrac{(\Delta P - \rho g l \sin\theta) D^2}{32 \mu l}$

$$Q = \frac{\pi (\Delta P - \rho g l \sin\theta) D^4}{128 \mu l}$$

* These expression can be derived directly from N-S equation as well.

$$\frac{\partial (\rho)}{\partial t} + \rho \vec{v} \cdot \nabla \vec{v} = -\nabla P + \rho \vec{g} + \mu \nabla^2 \vec{v}$$

$$\Rightarrow \mu \frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r} u\right) = \left(\frac{\partial P}{\partial x}\right) + \rho g \sin\theta$$

$u(r=0) = \text{finite}$

$u(r=R) = 0$ (No-slip)

(Do it at home !!)

* The above examples were nice demonstration that N-S and $\Sigma \vec{F} = m\vec{a}$ produces same results.

* What about dimensional analysis?

* For horizontal pipe flow the important parameters are ΔP, V, ℓ, D, μ (Density is not included, why!)

Thus, $K = 5$ and $r = 3$ (M, L & T)

* we need 2 π-terms [repeating variables are D, V and μ]

* $\pi_1 = \Delta P \, D^a V^b \mu^c \longrightarrow \pi_1 = \left(\frac{\Delta P D}{\mu V}\right)$

* $\pi_2 = \ell \, D^a V^b \mu^c \longrightarrow \pi_2 = \left(\frac{\ell}{D}\right)$

* Thus, $\left(\frac{\Delta P D}{\mu V}\right) = \phi\left(\frac{\ell}{D}\right)$

* What can we do now?
  - Assume using intuition.
    → ΔP is proportional to length (Linear)

* $\left(\frac{D \Delta P}{\mu V}\right) = \left(C/D\right) \ell \longrightarrow$ From Dimensional Analysis

Proportionality coefficient.

$\Rightarrow \left(\frac{\Delta P}{\ell}\right) = \left(\frac{C \mu V}{D^2}\right)$ or $V = \left(\frac{\Delta P D^2}{C \ell \mu}\right)$

* Flow rate $Q = \left(\frac{\pi D^4 \cdot \Delta P}{4 C \ell \mu}\right)$

$C = 32$ for circular pipe (Other values for other Geometries)

* Putting $C = 32$ in the expression for pressure drop we get,

$$\Delta P = \left(\frac{C l \mu v}{D^2}\right) = \left(\frac{32 \, l \mu v}{D^2}\right)$$

* Dividing by $\frac{1}{2}\rho v^2$ (Dynamic pressure)

$$\left(\frac{\Delta P}{\frac{1}{2}\rho v^2}\right) = \frac{32 \, l \mu v}{D^2 \cdot \frac{1}{2}\rho v^2} = \left(\frac{64}{Re}\right)\left(\frac{l}{D}\right)$$

$$\Rightarrow \quad \Delta P = f\left(\frac{l}{D}\right)\left(\frac{\rho v^2}{2}\right)$$

friction factor $\left(64/Re\right)$

* The pressure drop can be expressed as head (head loss),

$$h_{major} = \left(\frac{\Delta P}{\rho g}\right) = \left(\frac{f l v^2}{2 g D}\right)$$

$$\boxed{h_{major} = f\left(\frac{l}{D}\right)\left(\frac{v^2}{2g}\right)}$$

friction factor (64/Re)

Geometric ratio

Dynamic head

(Darcy - Weisbach equation)

* what happens if the flow is turbulent?

* Tube roughness can affect the flow (Pressure loss)
* Similar dimensional analysis for rough pipe (ε = tube/pipe roughness in meters) gives 6 variables (ΔP, v, l, D, ρ, μ and ε) and 3 dimensions (MLT). Thus, 4 π-terms are required.
* These 4 π-terms can be obtained as

$$\pi_1 = \left(\frac{\Delta P\, D}{\mu v}\right), \quad \pi_2 = \left(\frac{\rho v D}{\mu}\right), \quad \pi_3 = \left(\frac{l}{D}\right)$$

and $\pi_4 = \left(\frac{\varepsilon}{D}\right)$ (ε/D is called relative roughness)

which gives, $\left(\frac{\Delta P D}{\mu v}\right) = f\left(Re, \frac{l}{D}, \frac{\varepsilon}{D}\right)$

* We apply similar assumption that $\Delta P \propto l$

$$\left(\frac{\Delta P D}{\mu v}\right) = \frac{l}{D} f\left(Re, \frac{\varepsilon}{D}\right) = \left(\frac{l}{D}\right) C$$

$$\boxed{C = \phi\left(Re, \frac{\varepsilon}{D}\right)}$$

* Non-dimensionalization by dynamic pressure and rearrangement gives

$$\Delta P = f\left(\frac{l}{D}\right)\left(\frac{\rho v^2}{2}\right)$$

$f = \phi\left(Re, \varepsilon/D\right)$

* In terms of head loss,

$$\boxed{h_{major} = f\left(\frac{l}{D}\right)\left(\frac{v^2}{2g}\right)}$$

friction factor $\phi\left(Re, \frac{\varepsilon}{D}\right)$ — Geometric ratio — Dynamic head

Things to Remember

1. Major loss arises from 2 factors
   (a) Viscous loss (Internal friction)
   (b) Frictional loss (External friction)

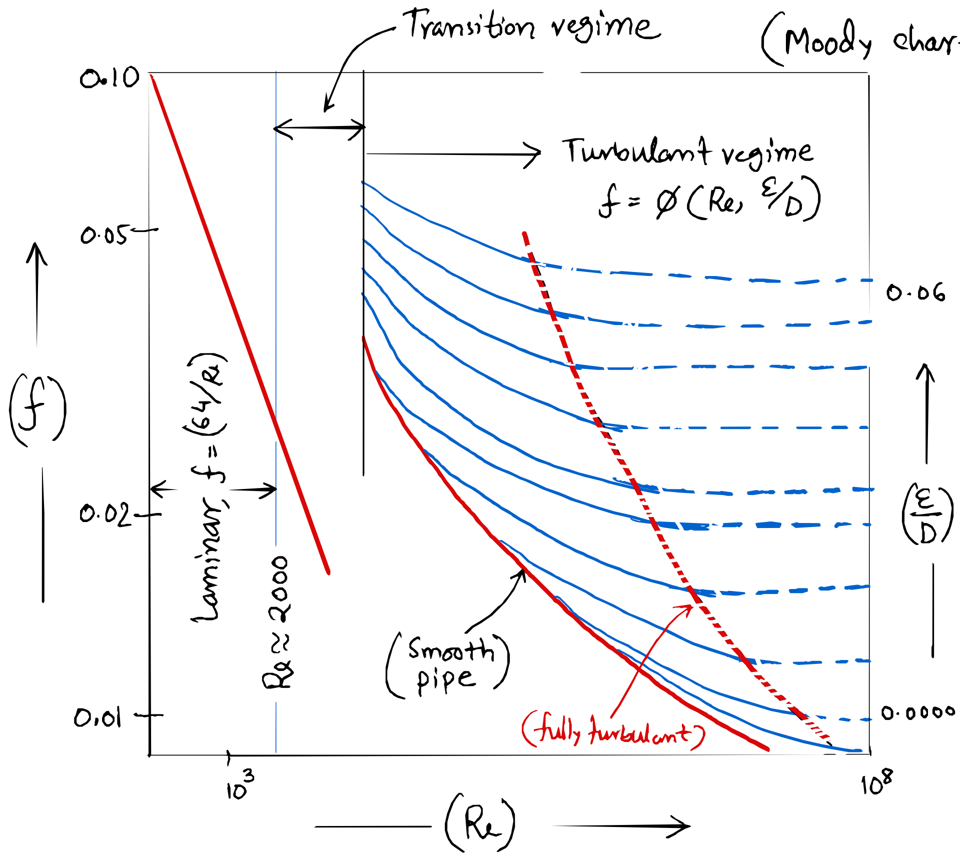
2. The general expression for major loss can be obtained as head loss

$$h_{major} = f\left(\frac{l}{D}\right)\left(\frac{v^2}{2g}\right)$$

→ $f = \left(64/Re\right)$ laminar

→ $f = \phi\left(Re, \varepsilon/D\right)$ Turbulent (Moody diagram)

(Moody chart)



* See figure 8.20 (Page-431) from textbook.

* Colebrook formula:

$$\frac{1}{\sqrt{f}} = -2 \log\left[\left(\frac{\varepsilon/D}{3.7}\right) + \left(\frac{2.51}{Re\sqrt{f}}\right)\right]$$

→ Implicit in nature (Needs trial & error)

→ Can use Haaland equation (10 % error)

$$\frac{1}{\sqrt{f}} = -1.8 \log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \left(\frac{6.9}{Re}\right)\right]$$

Minor losser

* losses due to fittings and acessories.

$$\text{fittings and acessories} \begin{cases} \rightarrow \text{Valves} \\ \rightarrow \text{Bends} \\ \rightarrow \text{Joints} \\ \rightarrow \text{Nozzles} \\ \rightarrow \text{Diffusers} \\ \rightarrow \text{Connectors} \end{cases}$$

* From dimensional analysis it is easi to show that for these fitting as well the loss can be expressed using Darcy-Weisback equation

$$h_{minor} = f\left(\frac{l}{D}\right)\left(\frac{v^2}{2g}\right)$$

* For different fittings it is very tough to determine l and D (usually they vary). Thus a single parameter is defind as

$$K_L = f\left(\frac{l}{D}\right) \rightarrow \left(\text{loss coefficient must be determined experimentally}\right)$$

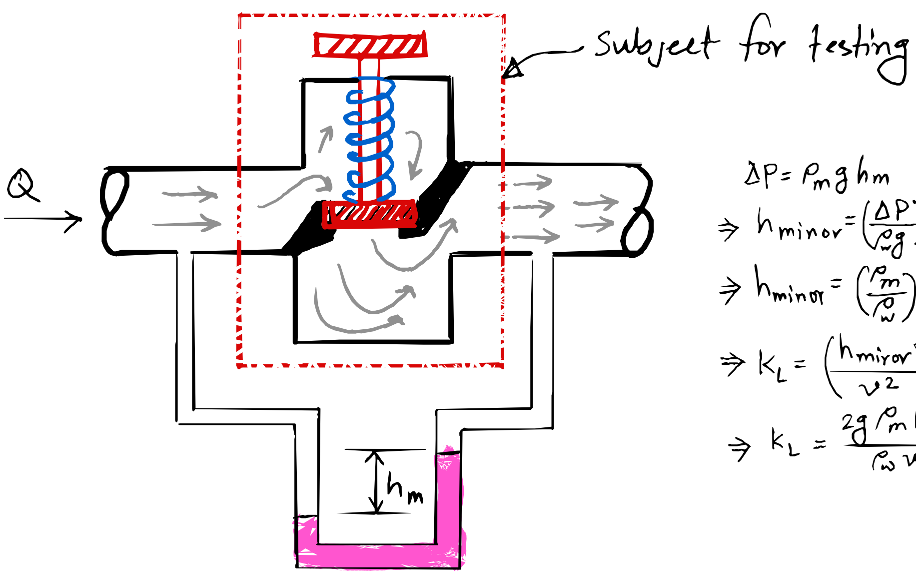
* $$h_{minor} = K_L\left(\frac{v^2}{2g}\right)$$

* Sometimes the loss coefficient is given as equivalent length (l_{eq}).

$$l_{eq} = \left(\frac{K_L D}{f}\right)$$

(Represents the length of pipe that would produce same loss.)

* K_L are usually provided from manufacturer or else need to be determined experimentally.



$\Delta P = \rho_m g h_m$

$\Rightarrow h_{minor} = \left(\frac{\Delta P}{\rho_w g}\right)$

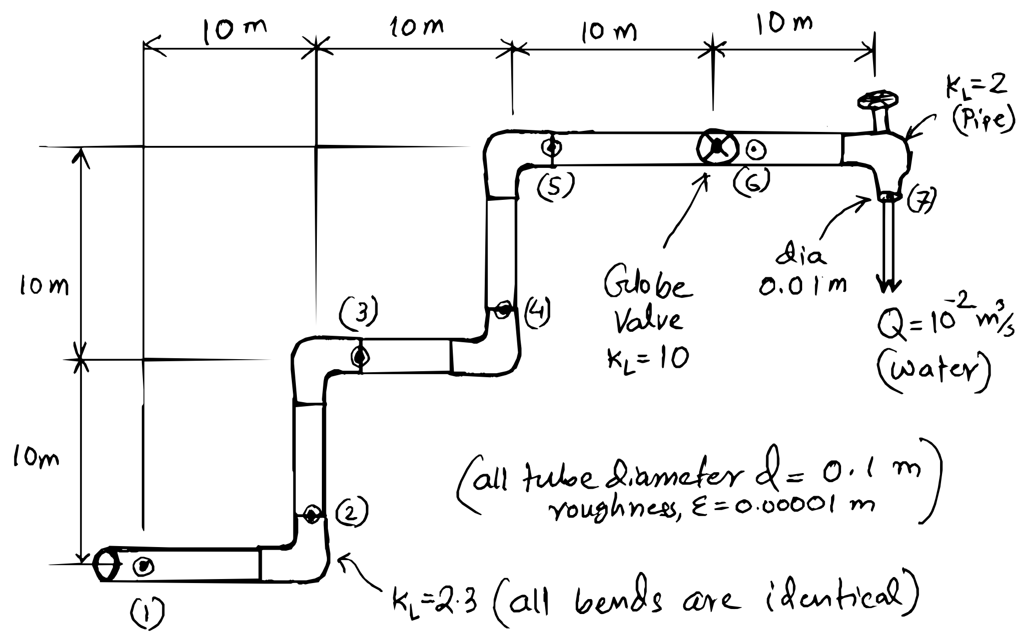
$\Rightarrow h_{minor} = \left(\frac{\rho_m}{\rho_w}\right) h_m$

$\Rightarrow K_L = \left(\frac{h_{minor}}{v^2}\right) 2g$

$\Rightarrow K_L = \frac{2g \rho_m h_m}{\rho_w v^2}$

* experimental setup for determining K_L (or l_{eq})

Example : Geometry known, determining loss or power requirement.



* what is the pressure at point ①, if

(a) all losses are neglected (No viscous loss + ideal fitting)

(b) Only major losses are present (ideal fittings)

(c) Both major and minor losses are present.

(a) No losses. Apply Bernoulli's equation between point ① and ⑦

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = P_7 + \frac{1}{2}\rho v_7^2 + \rho g z_7$$

* Here, $v_7 = (Q/A_7) = \left\{10^{-2} / \frac{\pi}{4}(0.01)^2\right\} = 127.3\ m/s$

* $v_1 = (Q/A_1) = \left\{10^{-2} / \pi/4 (0.1)^2\right\} = 1.27$ m/s

* $z_1 = 0$ (Datum reference), $z_7 = 20$ m

* $P_7 = 0$ (Gauge pressure)

Thus, $P_1 = \frac{1}{2}\rho(v_7^2 - v_1^2) + \rho g(z_7 - z_1)$

$\Rightarrow P_1 = 8297838.6$ Pa $[\approx 81.89$ atm (g)$]$

* We can also calculate pressure at each location as (all pressure gauge pressure)

$P_1 = 8297838.6$ Pa	$P_2 = P_1 = 8297838.6$ Pa
$P_3 = P_1 - 9800 \times 10 = 8199838.6$ Pa	$P_4 = P_3 = 8199838.6$ Pa
$P_5 = P_3 - 9800 \times 10 = 8101838.6$ Pa	$P_6 = P_5 = 8101838.6$ Pa
$P_7 = P_6 - \frac{1}{2}\rho(v_7^2 - v_6^2) = 0$ Pa	

(b) Major losses only (Ideal fittings $K_L = 0$)

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = P_7 + \frac{1}{2}\rho v_7^2 + \rho g z_7 + \Sigma(P_{major})$$

$$\Rightarrow P_1 = \underbrace{\frac{1}{2}\rho(v_7^2 - v_1^2)}_{\text{known}} + \underbrace{\rho g(z_7)}_{\text{known}} + \underbrace{\Sigma(P_{major})}_{\text{How?}}$$

$$* \sum P_{major} = \left(P_{major}^{1-2}\right) + \left(P_{major}^{2-3}\right) + \left(P_{major}^{3-4}\right) + \left(P_{major}^{4-5}\right) + \left(P_{major}^{5-7}\right)$$

$$P_{major}^{1-2} = \rho g h_{major}^{1-2} = \rho g \left[f \frac{l v^2}{2gd} \right]^{1-2}$$

* everything is known except f, which needs to be determined from Moody chart.

$$\rightarrow Re = \left(\frac{\rho v d}{\mu}\right) = \left(\frac{1000 \times 1.27 \times 0.05}{1.12 \times 10^{-3}}\right) = 56696.4$$

$\mu = 1.12 \times 10^{-3}$ (table 1.6)

$$\rightarrow (\varepsilon/d) = \left(\frac{0.00001}{0.1}\right) = 0.0001$$

$\rightarrow$ From moody diagram $f = 0.013$ (Appx)

* This value of f is same for all tube since the tube diameter doesnt change.

$$* \ P_{major}^{1-2} = \rho g f \left(\frac{l v^2}{2gd}\right) = 9800 \times 0.013 \times \left(\frac{10 \times 1.27^2}{2 \times 9.8 \times 0.1}\right)$$

$$\Rightarrow P_{major}^{1-2} = 1048.39 \ Pa$$

* Similarly for all tubes the major losses can be calculated as

$$P_{major}^{1-2} = 1048.39 \ Pa$$

$$P_{major}^{2-3} = P_{major}^{3-4} = P_{major}^{4-5} = 1048.39 \ Pa$$

$$P_{major}^{5-7} = 2 P_{major}^{5-6} = 2 P_{major}^{1-2} = 2096.77 \ Pa$$

$$\sum P_{major} = (4 \times 1048.39 + 2096.77) = 6290.33 \ Pa$$

Thus, $P_1 = \frac{1}{2}\rho\left(v_7^2 - v_1^2\right) + \rho g z_7 + \Sigma P_{major}$

$P_1 = 8304128.88\ Pa\ (\approx 81.96\ atm)$

Similarly we can calculate the pressures at each points

$P_1 = 8304128.88\ Pa$	$P_2 = P_1 - P_{major}^{1-2} = 8303080.49\ Pa$
$P_3 = P_2 - 9800\times10 - P_{major}^{2-3}$ $= 8204032.1\ Pa$	$P_4 = P_3 - P_{major}^{3-4} = 8202983.71\ Pa$
$P_5 = P_4 - 9800\times10 - P_{major}^{4-5}$ $= 8103935.32\ Pa$	$P_6 = P_5 - P_{major}^{5-6} = 8102886.93\ Pa$
$P_7 = P_6 - P_{major}^{6-7} - \frac{1}{2}\rho(v_7^2 - v_6^2)$ $= 0\ Pa$ (wow)	

(c) Both major and minor losses are present.

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1 = P_7 + \frac{1}{2}\rho v_7^2 + \rho g z_7 + \Sigma P_{major} + \Sigma P_{minor}$$

$$\Rightarrow P_1 = \underbrace{\frac{1}{2}\rho\left(v_7^2 - v_1^2\right)}_{\text{known}} + \underbrace{\rho g z}_{\text{known}} + \underbrace{\Sigma P_{major}}_{\text{known}} + \underbrace{\Sigma P_{minor}}_{\text{How?}}$$

$$\Sigma P_{minor} = P_{minor}^2 + P_{minor}^3 + P_{minor}^4 + P_{minor}^5 + P_{minor}^6 + P_{minor}^7$$

$$* \ P_{minor}^2 = \rho g K_L\left(\frac{v_2^2}{2g}\right) = 9800\times2.3\times\frac{(1.27)^2}{2\times9.8} = 1854.84\ Pa$$

$$* \ P_{minor}^3 = P_{minor}^4 = P_{minor}^5 = 1854.84\ Pa$$

* $P_{minor}^{6} = \rho g K_L\left(\frac{v_6^2}{2g}\right) = 9800 \times 10 \times \frac{(1.27)^2}{2 \times 9.8} = 8064.5\ Pa$

* $P_{minor}^{7} = \rho g K_L\left(\frac{v_6^2}{2g}\right) = 9800 \times 2 \times \frac{(1.27)^2}{2 \times 9.8} = 1612.9\ Pa$

* $\Sigma P_{minor} = (4 \times 1854.84) + 8064.5 + 1612.9 = 17096.76\ Pa$

* Thus, $P_1 = \frac{1}{2}\rho(v_7^2 - v_1^2) + \rho g z_7 + \Sigma P_{major} + \Sigma P_{minor}$

$\Rightarrow P_1 = 8321225.64\ Pa$ (≈ 82.12 atm)

* Similarly we can calculate pressures at each point as,

$P_1 = 8321225.64\ Pa$

$P_2 = P_1 - P_{major}^{1-2} - P_{minor}^{2} = 8318322.41\ Pa$

$P_3 = P_2 - 9800 \times 10 - P_{major}^{2-3} - P_{minor}^{3} = 8217419.18\ Pa$

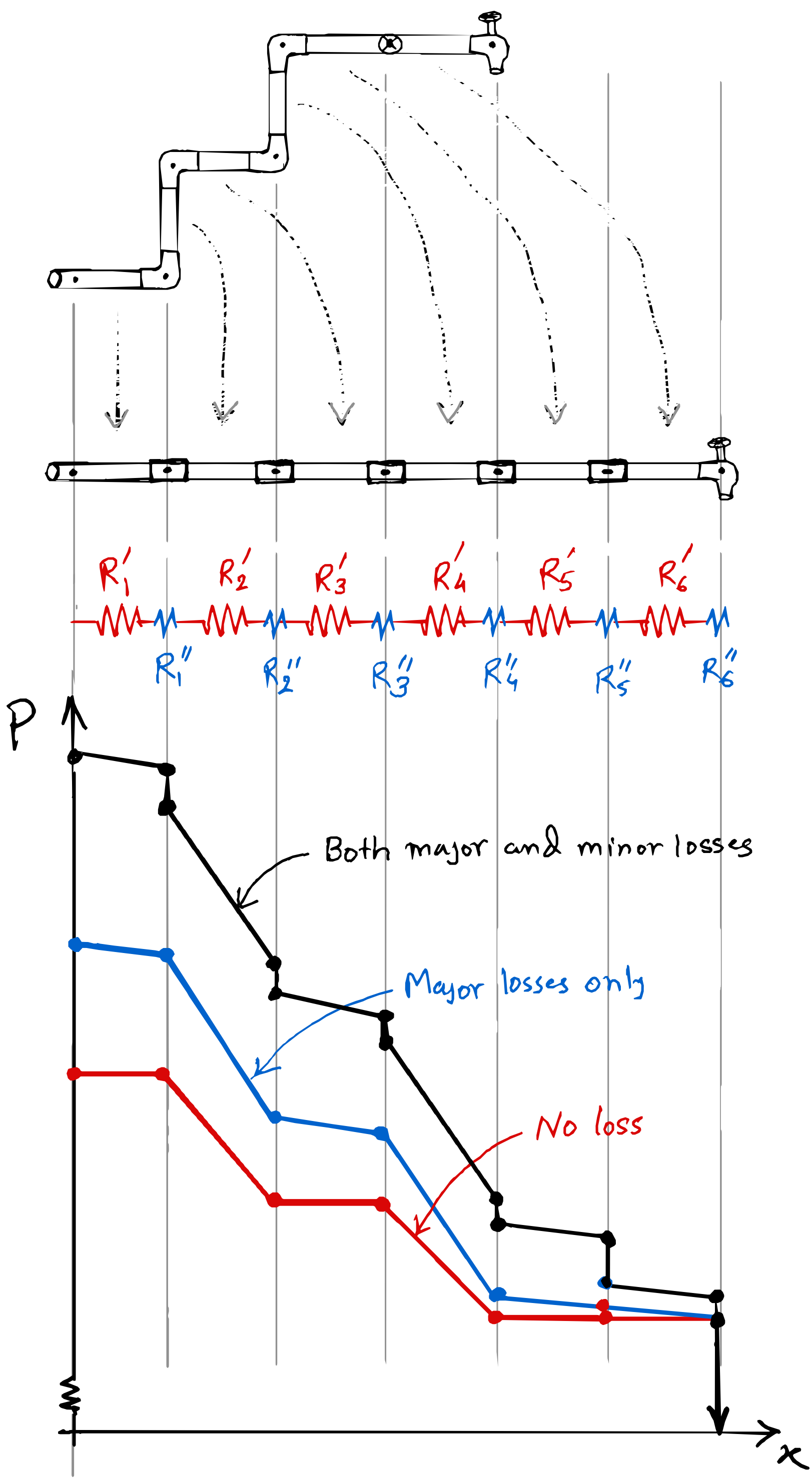
$P_4 = P_3 - P_{major}^{3-4} - P_{minor}^{4} = 8214515.95\ Pa$

$P_5 = P_4 - 9800 \times 10 - P_{major}^{4-5} - P_{minor}^{5} = 8113612.72\ Pa$

$P_6 = P_5 - P_{major}^{5-6} - P_{minor}^{6} = 8104499.83\ Pa$

$P_7 = P_6 - \frac{1}{2}\rho(v_7^2 - v_6^2) - P_{major}^{6-7} - P_{minor}^{7} = 0\ Pa$ (wow)

* The pressure variations are plotted in the figure.



R_1'
R_2'
R_3'
R_4'
R_5'
R_6'
R_1''
R_2''
R_3''
R_4''
R_5''
R_6''
P
Both major and minor losses
Major losses only
No loss
x

* We see that wee need more pressure at point 1 for losses.

** How much power a pump would take to raise this pressure? (inlet pressure to the pump is zero)

* $W_{pump} = Q\,\Delta P = Q\,(P_{out} - P_{in})$

$= 10^{-2}\left(\frac{m^3}{s}\right) \times 8321225.64 \left(\frac{J}{m^3}\right)$

$= 83212.25\ (J/s)$

$\approx 83.2\ kW$

$\approx 111.5\ hp$ (too big) $\boxed{1\ hp = 746\ W}$

* From only enery perspective (change in potential energy)

$W_{pump} = \dot{m}g(H) = \rho g Q H$

$= 9800 \times 10^{-2} \times 20$

$= 1960\ W$

$= 1.96\ kW\ (\approx 2.63\ hp)$

* Where is most pump power going? (Select from the options bellow)

(a) losses $(P_{loss} = P_{major} + P_{minor} = 6290.33 + 17096.76 = 23.3\ kPa$

(b) Kinetic energy of fluid @ outlet.

$P_{kinetic} = \frac{1}{2}\rho\,(v_7^2 - v_1^2) = 8101.8\ kPa$