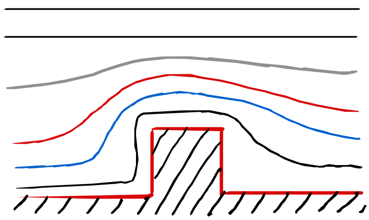
Ch-3

# Streamlines & acceleration

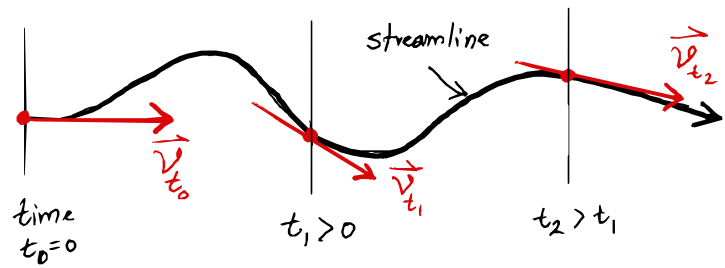
* What is a streamline?

① These are the lines that fluid particles follow.



See video (V-4/5)

② Complete picture is "line that is tangential to instantaneous velocity vector"

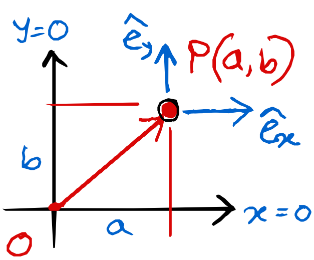



## Stream line co-ordinate system (2-D)

→ any 2-D co-ordinate system must have

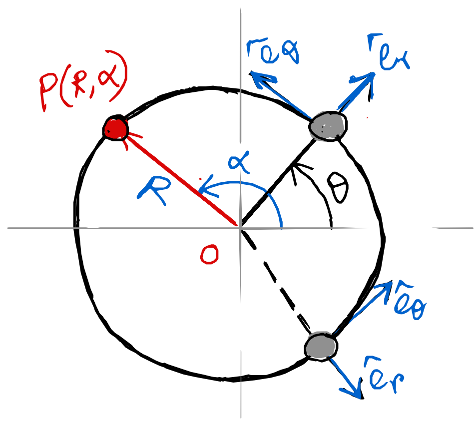
ⓐ 2 axis (direction of basis vectors) that are orthogonal (90° with one another)

ⓑ The 2 axes must be linearly independent




x axis (horizontal, →)
y axis (vertical ↑)

$$\vec{OP} = a\,\hat{e}_x + b\,\hat{e}_y$$




r axis (radia)

θ axis (circular)

$$\vec{OP} = R\,\hat{e}_r + \alpha\,\hat{e}_\theta$$

origin, r=0, θ=0

* $\hat{e}_r$ and $\hat{e}_\theta$ changes directions based on location.

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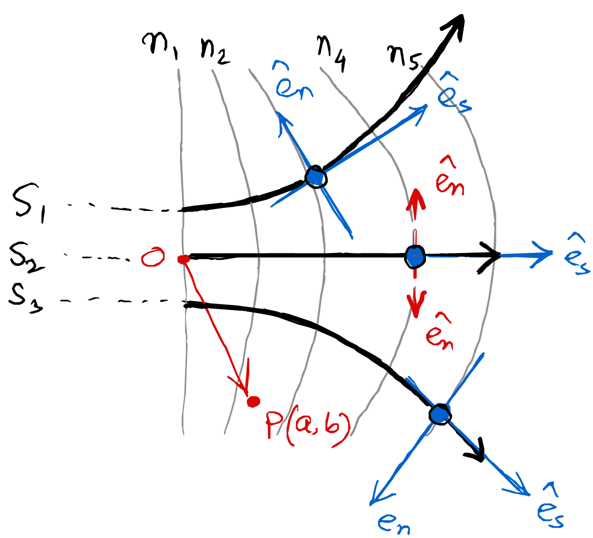



s-axis

n-axis

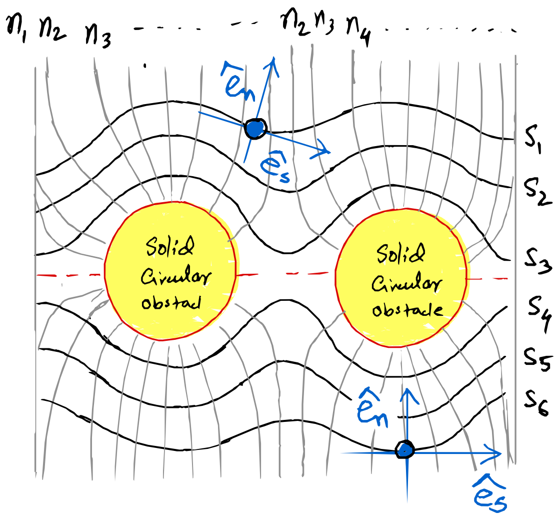
$$\vec{OP} = a\,\hat{e}_s + b\,\hat{e}_n$$

origin s=0, n=0

* a new co-ordinate system is designed.

* Stream lines can be extra ordinarily curvy.

→ Design an streamline coordinate system for the stream lines shown below:




where is the origin ??

→ wherever $s=0$ and $n=0$.

→ Can have multiple origin !!

* Since stream lines are always tangential to velocity vectors, No flwo occures across any streamlines.

* The normal velocity component is always zero in this co-ordinate representation.

* The 2-D flow becomes 1-D flow in this "frame of reference". (Interesting !!)

* Complex physic in simple frame of reference can be described as simple physics in complex frame of reference. (Complex does not mean imaginery)

$$\boxed{\begin{array}{c}\text{Complex physics}\\ \text{(Simple FOR)}\end{array}} \longrightarrow \boxed{\begin{array}{c}\text{Simple physics}\\ \text{(Complex FOR)}\end{array}}$$

math comes here

* FOR - frame of reference.

## * Acceleration along a stream line

$$\vec{a}_s = \frac{d\vec{v}}{dt} \longrightarrow a_s = \frac{\partial v}{\partial t}$$

$$\vec{v} = v\,\hat{e}_s \qquad \left(\begin{array}{l}\vec{v} = v_s\,\hat{e}_s + v_n\,\hat{e}_n\\ v_s = |\vec{v}| = v \;\; \& \;\; v_n = 0\end{array}\right)$$

$$\vec{a} = a_s\,\hat{e}_s$$

Thus, $a_s = \left(\frac{dv}{ds}\right)\left(\frac{ds}{dt}\right)$ * chain rule → think!!

$$\Rightarrow a_s = v\frac{dv}{ds}$$

* Does $v_n = 0$ mean $a_n = 0$?

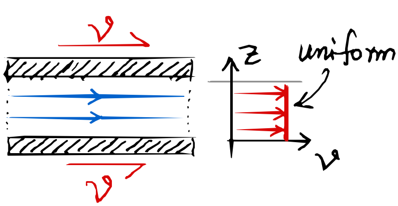
→ No, Though $v_n = 0$, $V_s$ is not always straight. Thus centripetal force (acceleration) is present. (We will prove this in Ch-4)

* $a_n = \dfrac{v^2}{R_c}$ ($R_c$ = radius of curvature)

## Summery

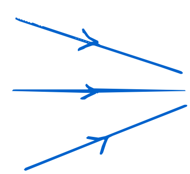
* $$\left\{\begin{array}{l} a_s = v\left(\dfrac{\partial v}{\partial s}\right) \\[2ex] a_n = \left(\dfrac{v^2}{R_c}\right) \end{array}\right\}$$ acceleration expressions in stream line coordinate

* Always remmumber: whereever there is acceleration there is a force.



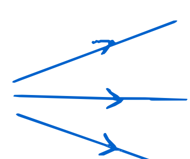

$a_s = 0$, $a_n = 0$ $\quad \mu = 0$

$\mu = 0$



$a_s > 0$

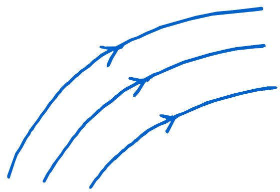
$a_n = 0$



$a_s < 0$

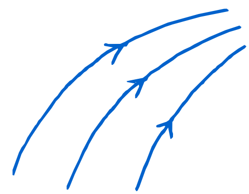
$a_n = 0$

$\mu = 0$



$a_n > 0$

what about $a_s$ ?



$a_s > 0$, $a_n > 0$

* Is it possible to have $a_n < 0$ ??

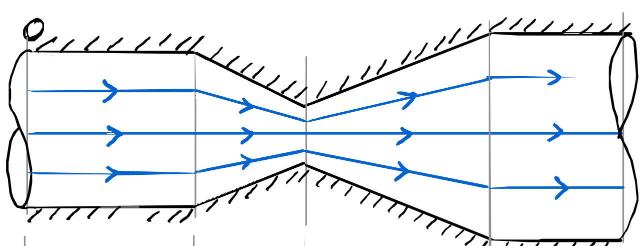
* Case study (C-D Nozzle) → Simplified

(a) No viscosity ($\mu = 0$)

(b) No curved stream lines. $a_n = 0$

$a_n = 0$

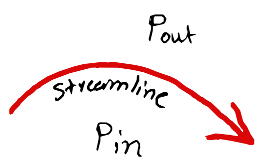
No curved streamline



| | | | | |
|---|---|---|---|---|
| Acceleration: | $a_s = 0$ | $a_s > 0$ | $a_s < 0$ | $a_s = 0$ |
| Force: | 0 | → | ← | 0 |

* If $v$ changes on streamlines then there is an accelaration !!

* When stream line curves there is always a non zero (>0) centripetal acceleration (centripetal force).
  → How do fluid particles (lets say molecules) understant this force?
  → Pressure !!!
  (Details later)




$P_{out} > P_{in}$

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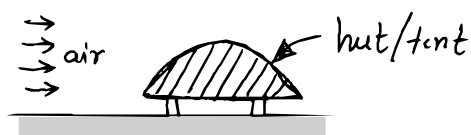
* why air flow causes hut/tent to go up??

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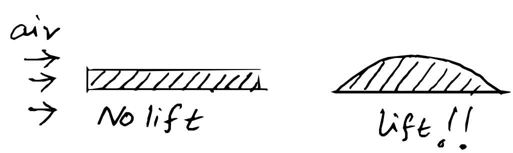
* Think: how airplane wings genrate lifting force? what can you do to increase lifting force?



# $\sum F = ma$ along a streamline

* We know acceleration in streamline co-ordinate becomes $a_s = v\left(\frac{dv}{ds}\right)$.

* Also the pressure field is governed by

$$\underbrace{-\nabla P + \rho\vec{g}}_{\text{Forces}} = \underbrace{\rho\vec{a}}_{\text{Inertia}} \longrightarrow \text{(Vector equation)}$$

* Now we decompose the above eqn into the S-component (in streamline co-ordinate).

$\rightarrow$ S-component: $-\frac{\partial P}{\partial s} + \rho g_s = \rho\left(v\frac{dv}{ds}\right)$

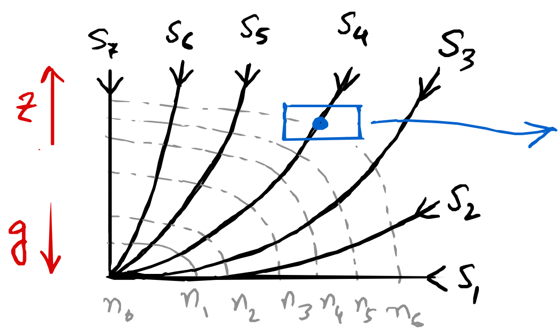
* Here we can apply a trick, $v\frac{\partial v}{\partial s} = \frac{\partial}{\partial s}\left(\frac{1}{2}v^2\right)$
"try this yourself"

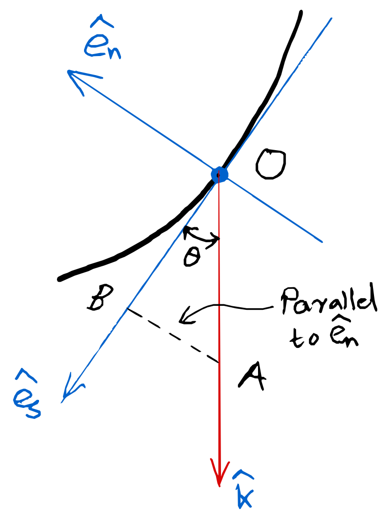
* Substitution $\longrightarrow$ $-\frac{\partial P}{\partial s} + \rho g_s = \frac{\rho}{2}\left(\frac{\partial v^2}{\partial s}\right)$

* What about $g_s$ ??
  $\rightarrow$ We know it works vertically downward. which is a non-changing direction while $\vec{s}$ can change any way, based on location in the frame of reference.

  $\rightarrow$ $g_s$ = Streamwise component of $g$.

* Consider an arbitrary stream line co-ordinate system.




* Component rule:

$$g_s = g \cdot \cos\theta$$

$\rightarrow$ again, $\cos\theta = \dfrac{OA}{OB} = \dfrac{\partial z}{\partial s}$

* Thus, $g_s = -g \dfrac{\partial z}{\partial s}$ {make sure you understand this. Not hard/tough}

* If 's' does not changes direction and is vertical (upward) $\dfrac{\partial z}{\partial s} = \pm 1$

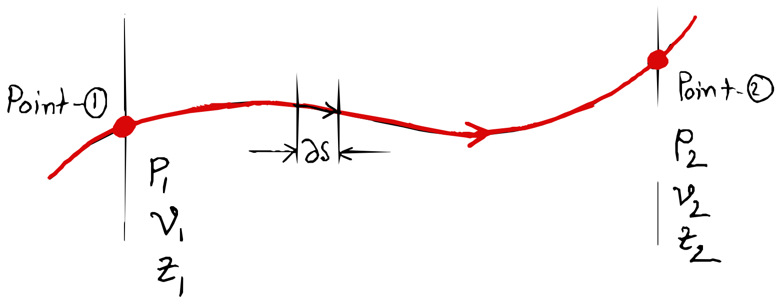
* If $\hat{e}_s$ does not changes direction and is horizontal then z is always allined with $\hat{e}_n$. Thus $z \neq f(s)$ and $\dfrac{\partial z}{\partial s} = 0$.

* Now we have all components known to apply $\sum F = ma$ along s-direction.

* Substitution gives

$$-\left(\frac{\partial P}{\partial s}\right) - \rho g\left(\frac{\partial z}{\partial s}\right) = \frac{\rho}{2}\left(\frac{\partial v^2}{\partial s}\right)$$

* Integrating between two points on a single streamline gives,




$$\int_{P_1}^{P_2} dP + \rho g \int_{z_1}^{z_2} dz + \frac{\rho}{2}\int_{v_1}^{v_2} dv^2 = 0$$

$$\Rightarrow (P_2 - P_1) + \rho g (z_2 - z_1) + \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = 0$$

$$\Rightarrow P_2 + \frac{1}{2}\rho v_2^2 + \rho g z_2 = P_1 + \frac{1}{2}\rho v_1^2 + \rho g z_1$$

* Integrating $-\frac{\partial P}{\partial s} - \rho g \frac{\partial z}{\partial s} = \frac{\rho}{2}\left(\frac{\partial v^2}{\partial s}\right)$ between any 2-points shows that $\left(P + \frac{1}{2}\rho v^2 + \rho g z\right)$ must be identical for any points on a single stream line. Thus,

$$\boxed{P + \frac{1}{2}\rho v^2 + \rho g z = \text{Constant}}$$

* This equation is known as <u>Bernoulli's Equation</u>.

## Some comments on Bernoulli's equation

1. This equation has many-many limitations.
2. Albait it's limitation, this equation can be used to "solve" many practical/engineering "problems" with reasonable accuracy.
3. This equation is so famous, if you say, "I have fluid mechanics knowledge" Probably you are going to be asked about this equation next.

# Limitations of Bernoulli's equation

① Viscous effect is negligible. Does not mean that the fluid needs to have zero viscosity.

→ Remmember all fluid has viscosity, which is a "dynamic proparty" meaning it only acts when fluid moves.

→ If the other forcess (causing fluid flow) are larger compared to the shear stess developed then the flow can be assumed to have negligible viscous effect.

* Usually defined by Reynold's number (Re).

② Steady flow. $\left[\text{Mean } \frac{\partial}{\partial t}(\,) = 0\right]$

→ what does "steady flow" mean

opposite meaning !!!

* See video (V-6) of laminar flow

③ Incompressible flow. (Does not mean incompressible fluid)

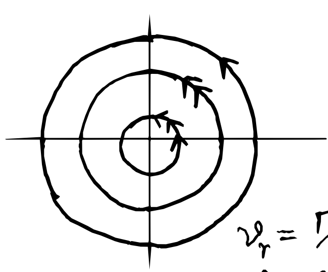
→ A compressible fluid can have an incompressible flow, and similarly an incompressible fluid can have a compressible flow.

* It is almost always defined by Mach number (Ma)

④ Irrotational flow. Does not mean angular velocity $\omega = 0$.

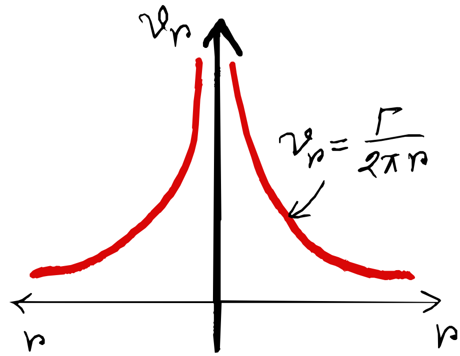
→ Movement does not have any rotationality. (See video - 7/8/9)

→ Pure vortex flow is an irrotation flow.

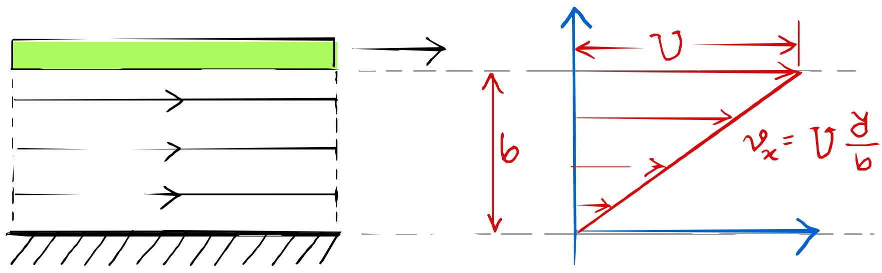


$v_r = \Gamma / 2\pi r$

$v_\theta = 0$




irrotational vortex



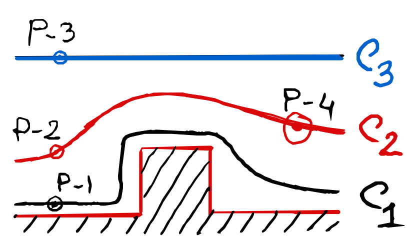

Rotational flow

⑤ Bernoulli's equation must be applied along a single stream line (if the flow is rotational).

(*) What changes among stream lines?

→ why Bernoulli's equation can not be applied across stream lines?

* we know that when fluid moves, its particles (molecules) carries energy.




Since no flow occures across any stream lines it can not be said that the total energy between two particles on separate stream lines are same, (eg. $TE_{P-4} = TE_{P-2} \neq TE_{P-2} \neq TE_{P-3}$)

* In other word: The coefficient appearing in Bernoulli's equation (on Right hand side) are not same for different stream lines.

# $\sum F = ma$ across streamline

* Why not? Lets see what can happen!!

* Basic pressure field equation:

$$-\nabla P + \rho \vec{g} = \rho \vec{a}$$

* Along $\hat{n}$ direction, $-\frac{\partial P}{\partial n} - \rho g_n = \rho\left(\frac{v^2}{r}\right)$

* In a similar way we can show that

$$g_n = -g\frac{\partial z}{\partial n} \qquad \left[\text{previously used} \rightarrow g_s = -g\frac{\partial z}{\partial s}\right]$$

Thus,

$$\boxed{-\underbrace{\frac{\partial P}{\partial n}}_{\left(\text{Pressure gradient in n-dir.}\right)} - \underbrace{\rho g\frac{\partial z}{\partial n}}_{\left(\text{Particle weight variation in n-dir}\right)} = \underbrace{\rho\left(\frac{v^2}{r}\right)}_{\left(\text{If } r \neq \infty \text{ Centripetal force}\right)}}$$

* Integration gives:

$$\boxed{P + \rho\int\left(\frac{v^2}{r}\right)dn + \rho g z = \text{Constant.}}$$

(Not very use full if $r$ is known precisely)

* If flow is in horizontal plane ($\perp$ to $z$) then $\frac{\partial z}{\partial n} = 0$, which gives

$$\frac{\partial P}{\partial n} = -\left(\frac{\rho v^2}{r}\right)$$

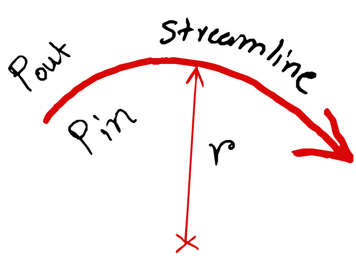
* What does it imply?

$\rightarrow$ if $v \neq 0$ and $r \neq \infty$ then $\frac{\partial P}{\partial n} \neq 0$.

(a) pressure changes in $\hat{n}$ direction.

(b) If $r$ increases pressure difference decreases.

(c) Pressure difference is in opposite direction to $r$.




$$\boxed{P_{out} > P_{in}}$$

## Bernoulli's equation (Pressure Interpretation)

ⓐ what are the units of 3 terms appearing in the equation:

$$P + \frac{1}{2}\rho v^2 + \rho g z = \text{Constant}$$

(Pa) (Pa) (Pa) (Pa)

dimensional homogenity !!!

→ Each term represents some kind of pressure.

→ The first term is called static pressure.

→ Second term is called Dynamic pressure (caused by velocity!!)

→ Third term is called hydrostatic pressure (caused by gravity!!)

## * Stagnation pressure

→ When fluid flow is stopped (Remmember valve closing to understand acustic speed), what would happen to the kinetic energy?

$$* \quad \underbrace{\frac{1}{2}\rho v^2}_{} = \frac{1}{2}\frac{m}{\forall}v^2 = \frac{1}{\forall}\left(\frac{1}{2}mv^2\right)$$

→ (Kinetic energy per unit fluid volume !!!)

Case study: Jet on a vertical wall.

* apply Bernoulli's eqn

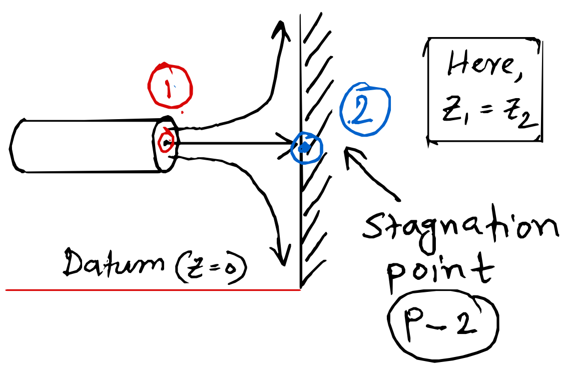
$P_1 = P_1$ (known)

$v_1 = v$ (known)

$z_1 = z_2$ (same elevation)

$P_2 = ?$ (unknown)

$v_2 = 0$ (stagnant)




$$P_2 + \cancelto{0}{\frac{1}{2}\rho v_2^2} + \cancel{\rho g z_2} = P_1 + \frac{1}{2}\rho v_1^2 + \cancel{\rho g z_1}$$

* $P_2 = P_1 + \frac{1}{2}\rho v_1^2$

$\rightarrow$ $v_1$ vanishes and increase pressure.

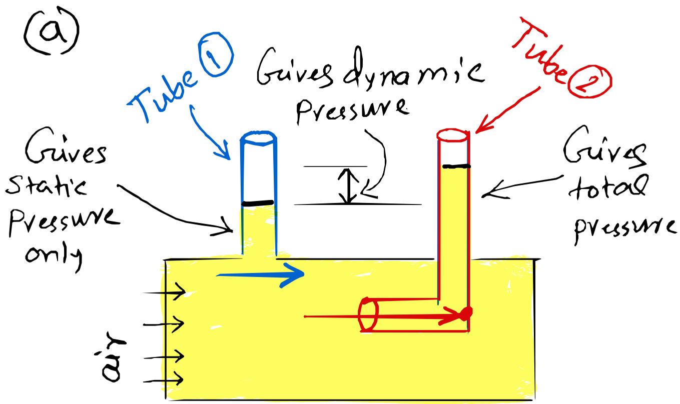
* The sum of static pressure and the dynamic pressure is called total pressure.

$$\begin{pmatrix}\text{Total}\\ \text{Pressure}\end{pmatrix} = \begin{pmatrix}\text{static}\\ \text{Pressure}\end{pmatrix} + \begin{pmatrix}\text{dynamic}\\ \text{pressure}\end{pmatrix}$$

## Velocity measurement

* How to measure velocity of invisible fluid?
* How to measure velocity of internal flow?

(a)




* $\rho_{air}\, g H = \frac{1}{2} \rho_{air} v^2$

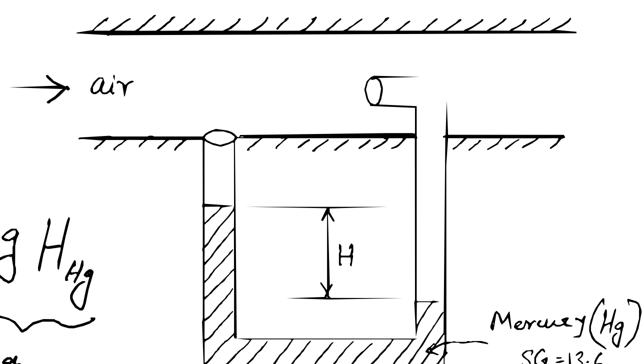
$\Rightarrow v = \sqrt{2gH}$.

* Tube (1), Gives the static pressure of the flowing air. This tube is called static tube.

* Tube (2), Gives the stagnation pressure and is called pitot tube.

# Pitot - Static tube

* why not use the static tube and the pitot tube to form a mannometer.

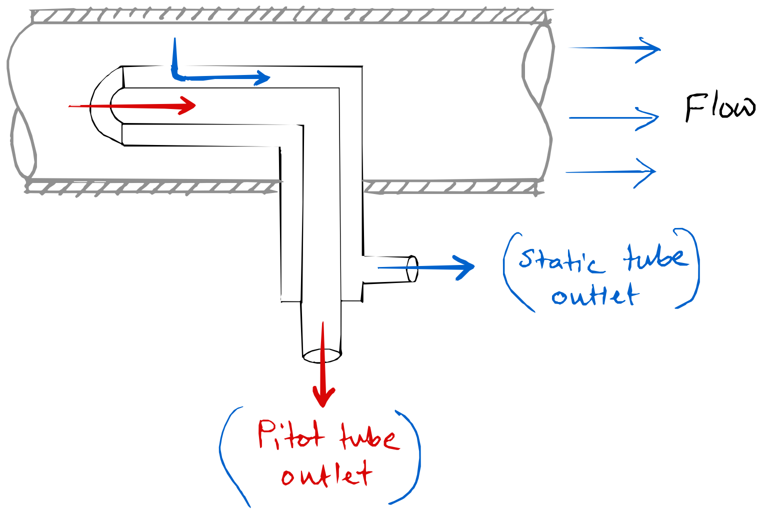



* again,

$$\underbrace{\frac{1}{2}\rho_{air} v_{air}^2}_{\substack{\text{air} \\ \text{(flowing fluid)}}} = \underbrace{\rho_{Hg}\, g\, H_{Hg}}_{\substack{\text{Hg.} \\ \text{(mannometeric fluid)}}}$$

$$\Rightarrow \quad v_{air} = \sqrt{2gH\left(\rho_{Hg}/\rho_{air}\right)}$$

* Pitot-static tube occationally comes as a single tube (looks like) with two tubes to be connected to the mannometer.




* Draw the pressure vs distance (along flow) plot for pitot-static tube insert.

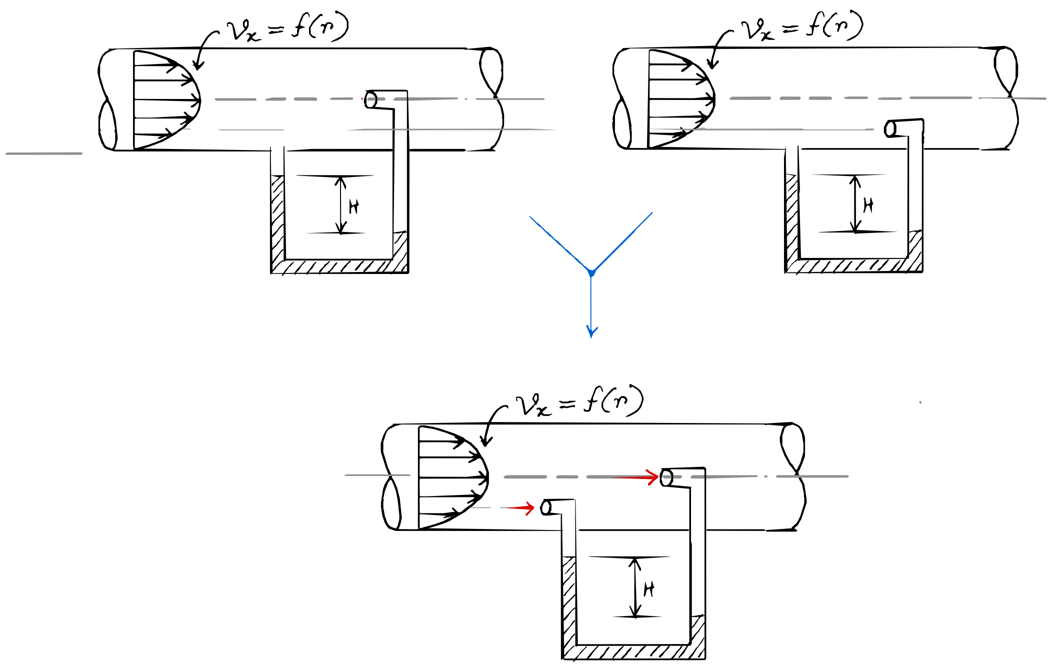
  → See figure 3.9 (text book)

* Design of pitot-static tube.

  → See figure 3.9 (text book)

# Intrussive flow measurement

* Basically velocity measurement. $Q = A_{cross} v$.
* Problem with pitot-static tube
  → Where to place the tube? Dependence of reading on radial location.



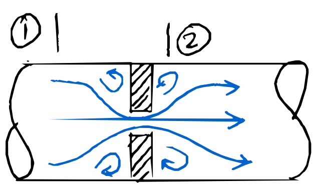

* Pitot-static tube are good for local velocity measurement. But for large-scale flow measurement we need something to measure average (mean flow rate).

# Average flow measurement

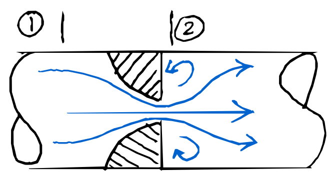
* Challenge → How to measure average velocity without measuring local velocities??

* Need something to convert velocity into pressure (average, Not local).

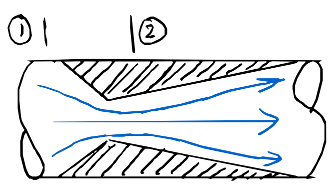
* 3-types of flow-meters are use often that operates on Bernoulli equation.




Orifice

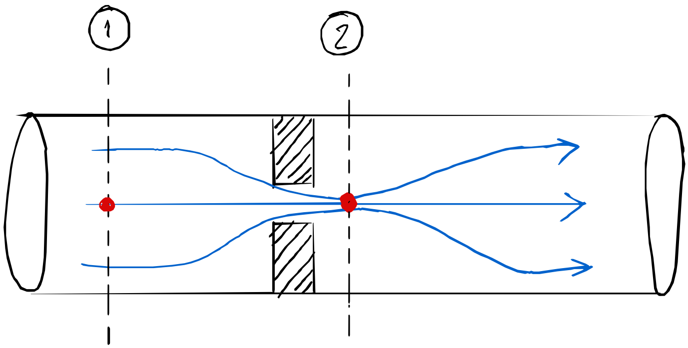



nozzle




Ventury

* The pressure difference between point 1 and 2 Gives the average velocity.




* applying Bernoulli equation between point 1 and 2 we get,

$$P_2 + \frac{1}{2}\rho v_2^2 = P_1 + \frac{1}{2}\rho v_1^2 \quad (z \text{ is neglected})$$

$$\Rightarrow \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = \left(P_1 - P_2\right)$$

$$\Rightarrow v_2^2 = v_1^2 + \frac{2(P_1 - P_2)}{\rho}$$

* From mass balance,

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \quad (\rho_1 = \rho_2 \text{ incomp.})$$

$$\Rightarrow v_1 = v_2\left(\frac{A_2}{A_1}\right)$$

So, $$v_2^2 = v_2^2\left(\frac{A_2}{A_1}\right)^2 + \frac{2(P_1 - P_2)}{\rho}$$

$$\Rightarrow \quad v_2^2\left[1-\left(A_2/A_1\right)^2\right] = \frac{2(P_2-P_1)}{\rho}$$

$$\Rightarrow \quad v_2 = \sqrt{\frac{2(P_2-P_1)}{\rho\left[1-\left(A_2/A_1\right)^2\right]}}$$

Thus, $Q = A_2 v_2$

$$\Rightarrow Q = A_2 \sqrt{\frac{2(P_2-P_1)}{\rho\left[1-\left(A_2/A_1\right)^2\right]}}$$

ⓐ $P_1 - P_2$ is measured

ⓑ $A_1$ & $A_2$ are known

ⓒ $\rho$ is the density of flowing fluid.

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* The above flow rate is the theoritical flow rate. Sometimes a correction factor is used (Impericaly obtained)

$$Q_{actual} = \underline{C_d} \cdot Q_{theoritical}$$

→ Coefficient of discharge (unitless).

# Bernoulli equation (Energy interpretation)

$$P + \frac{1}{2}\rho v^2 + \rho g z = \text{constant}$$

→ Each term has unit of Pa → $\left(\frac{J}{m3}\right)$

→ Thus each tem represent some sort of energy per unit fluid volume.

→ For volume $\forall$ this equation can be written as

$$P\forall + \frac{1}{2}\rho\forall v^2 + \rho\forall g z = \text{constant}$$

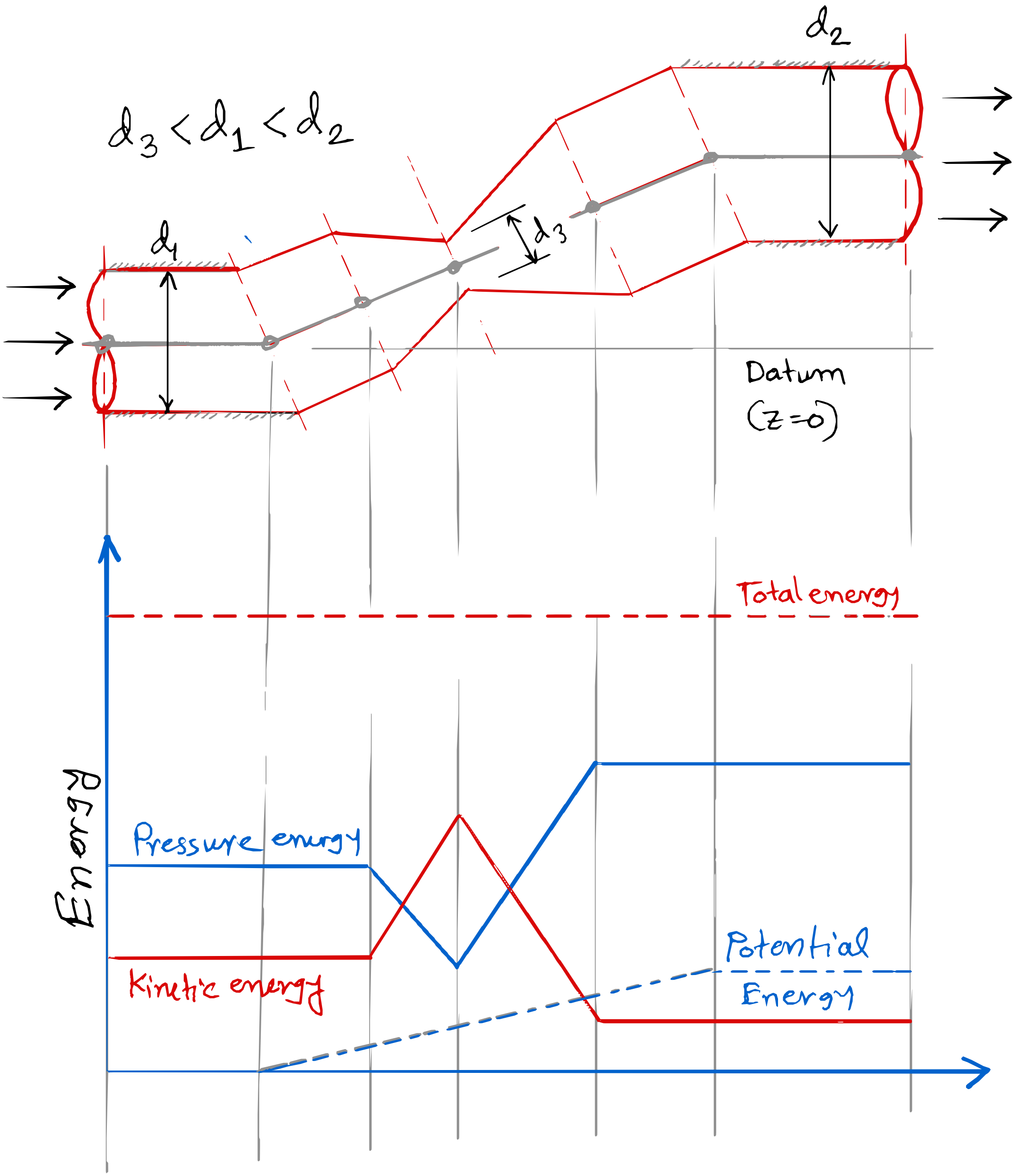
$$\Rightarrow P\forall + \frac{1}{2}mv^2 + mgz = \text{constant}$$

(Due to pressure) $\boxed{\text{Pressure energy}}$ ↑ $P\forall$ — Kinetic energy ↑ $\frac{1}{2}mv^2$ — Potential energy ↑ $mgz$ (Due to elevation)

* The term "pressure energy" is not well established. Rather it is considered a form of potential energy.

* Draw the energy lines for flow of water throug an inclined long Nozzle.

* Neglect frictional losses.



# Bernoulli's equation (Head Interpretation)

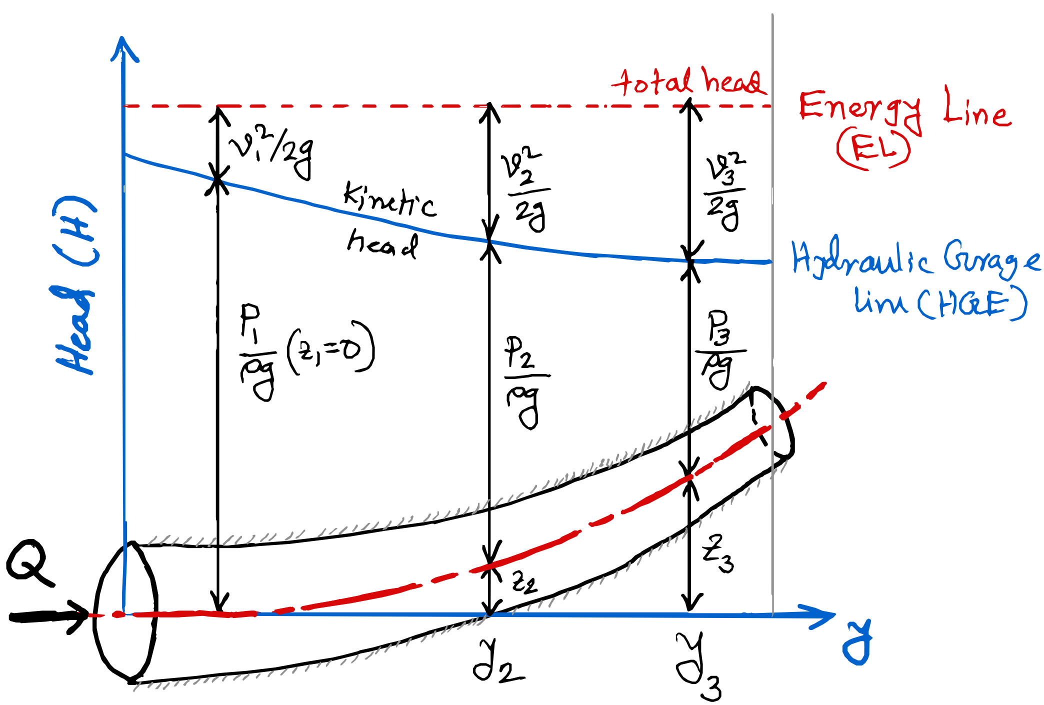
* We know any pressure can be converted into head of any fluid. (we do flowing fluid)

$$\left(\frac{P}{\rho g}\right) + \left(\frac{v^2}{2g}\right) + z = \text{Constant}$$

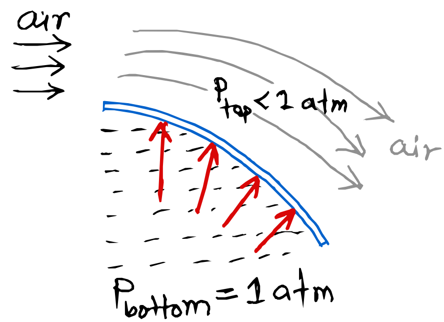
$\frac{P}{\rho g}$ → (Static pressure head)

$\frac{v^2}{2g}$ → (Dynamic pressure head) or (Kinetic head)

$z$ → (Datum head)



* Air blowing over a paper (class demonstration)



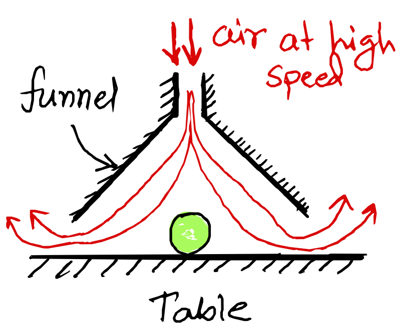

→ what happens?

→ why happens?

* See video on blowing air in between two bowling balls (Video-10)

(Explain it to a <5th Grader at home)

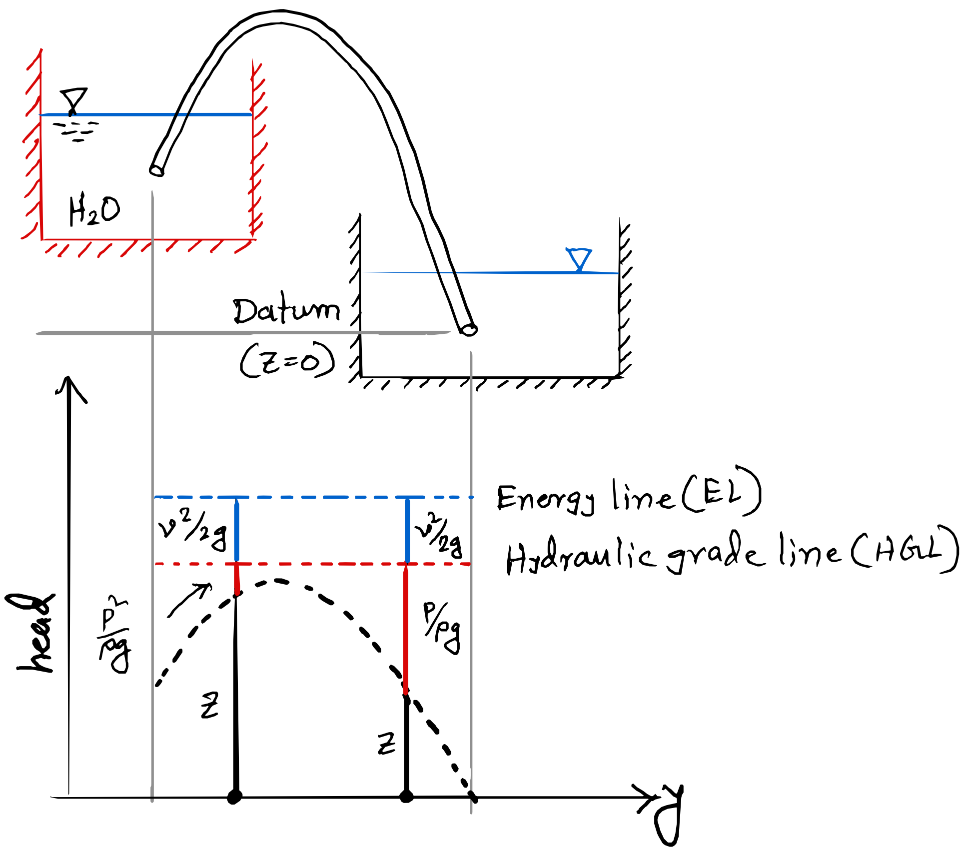
* what will happen in the folling case?




* See video on youtube at home (Video-11)

## * Siphon (The "Devil's trick")

→ See video-12




ⓐ Velocity does not change.

ⓑ Pressure is minimum at the hight point.

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* What will happen if someone cut the tube at the height point while water is flowing?

See video (V-13/14/15)

# Restrictions of BE

* Read section 3.8 yourself
* Revisit the limitations of Bernoulli's equation (BE).

Acronym for Bernoulli's equation.