Ch-1

* Fluid Vs Solid

ⓐ Different molecular structure (Not very useful)

ⓑ They behave (Deforms) differently upon applied shear stress (Force/Area)

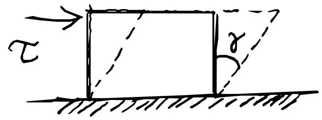
Fluid : Deforms contineously under applied shear stress (Dose not matter how small the shear stress is)

Solid : Deformation is proportional to the applied stress.

Solids

$$\sigma \propto \varepsilon \Rightarrow \sigma = \underset{\uparrow}{E}\,\varepsilon$$

Youngs Modulus (Elasticity)

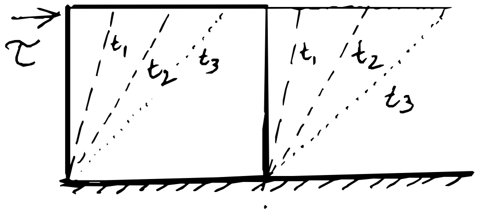


$$\tau \propto \gamma$$

$$\Rightarrow \tau = \underset{\uparrow}{G}\,\gamma$$

"Shear Modulus" (Rigidity)

Fluids



It flows. !!!

- * what could be a proper constitutive relationship?
- * Does "time" matters here?
- * How can we convert any quantity into time dependent quantity?

* Dimensions and units

(a)

Primary quantities
(4 in fluid dynamics)

Quantity	Unit	Dimension
Mass	kg	M
Distance	m	L
Time	s	T
Temperature	K	Θ

(b)

Secondary quantities
(unlimited)

Quantity	Unit	Dimension
Velocity	m/s	LT^{-1}
acceleration	m/s^2	LT^{-2}
Force	N (kgm/s^2)	MLT^{-2}
Pressure	Pa ($kg/m s^2$)	$ML^{-1}T^{-2}$

Quantity	Unit	Dimension
Force	N	F
Distance	m	L
Time	s	T
Temperature	K	Θ

Quantity	Unit	Dimension
Velocity	m/s	LT^{-1}
acceleration	m/s^2	LT^{-2}
Mass	Ns^2/m	$FL^{-1}T^2$
Pressure	N/m^2	FL^{-2}

See table 1.1 For more quantities

* Many systems of units are used:
(Mostly based on location, countries and Practices)

** SI - system (International)
British-system (FPS)

* Units and dimensions can be written in many forms.

Example:

* unit of gravitational acceleration "g".

(a) $g \rightarrow m/s^2 \rightarrow (m/s)/s$

* signifies rate of change of velocity for a free-falling object.

(b) $g \rightarrow \frac{N}{kg}$

* signifies amount of Force acting on an object per unit mass.

* Unit of velocity "v"

(a) $v \rightarrow m/s$

* signifies amount of distance traveled in unit time. (rate of displacement)

(b) $v \rightarrow \frac{p}{m}$ → momentum $\rightarrow \frac{kg(m/s)}{kg}$

* signifies momentum of unit mass.

(we will talk about this later in viscous flow)

Dimensional homogenity

* Physically different parammeters can-not be added or subtructed.

eg: 5 pens + 3 pens = 8 pens (ok)

5 apples + 3 oranges = ?? (Not ok)

* Different parameters can be multiplied and/or divided with/by to create new parameters.

eg: $$\underset{\substack{\uparrow \\ \text{New parameter}}}{\text{momentum}(P)} = \underbrace{\text{mass}\,(m) \cdot \text{velocity}\,(v)}_{\text{dis-similar parameter}}$$

* Any equation describing physical system must have dimensional homogenuty.

eg: $$\underset{\swarrow\,(m)}{S} = \underbrace{ut}_{\downarrow\,(m)} + \frac{1}{2}\underbrace{at^2}_{\downarrow\,(m)}, \qquad \underset{\swarrow\,(J)}{E} = \underbrace{mc^2}_{\searrow\,(J)}$$

Fluid mass & volumes

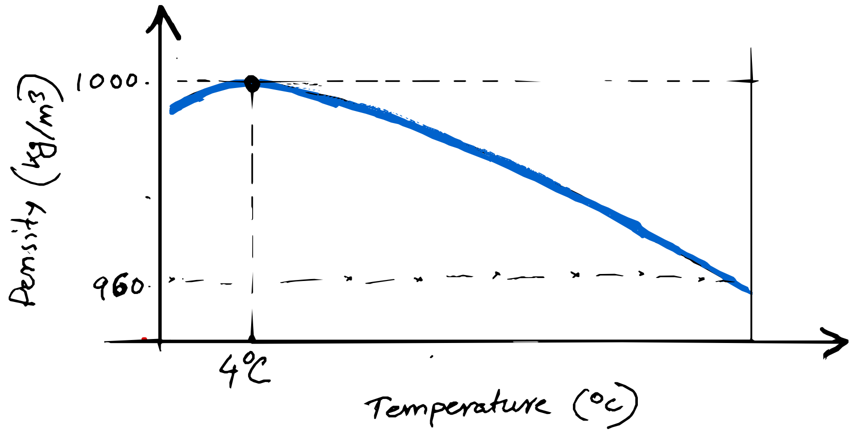
① Density, $\rho = \left(\frac{mass}{Volume}\right)$ $\begin{cases} \text{unit} \rightarrow kg\,m^{-3} \\ \text{dimension} \rightarrow ML^{-3} \end{cases}$

② Specific volume, $\nu = \left(\frac{volume}{mass}\right) = \frac{1}{\rho}$ $\begin{cases} \text{unit} \rightarrow m^3/kg \\ \text{dimension} \rightarrow L^3/M \end{cases}$

③ Specific weight, $\gamma = \rho g = \left(\frac{weight}{volume}\right)$ $\{ \text{unit} \rightarrow \frac{N}{m^3}$

④ Specific gravity, $SG = \left(\frac{\rho}{\rho_{H_2O}\big|_{4°c}}\right)$ $\{ \text{unitless}$

(why ρ_{H_2O} is taken at 4°C)



* what is the specific gravity of Mercury?
($\rho_{Hg} = 13600$ kg/m³)

* will it sink/float in water?

* Fluid properties and their units

1) mass : unit → kg

2) volume: unit → m^3

3) Density: unit → kg/m^3

4) Pressure: unit → Pa → N/m^2 **

5) Viscosity: unit → Pa·S → Ns/m^2

6) Surface tension: unit → N/m

* Other quantities & their units

1) Velocity : unit → m/s

2) Acceleration: unit → m/s^2

3) Force : unit → N (kgm/s^2)

4) stress : unit → N/m^2 (Pa) → (Different than pressure)

* What are the units of derivatives?

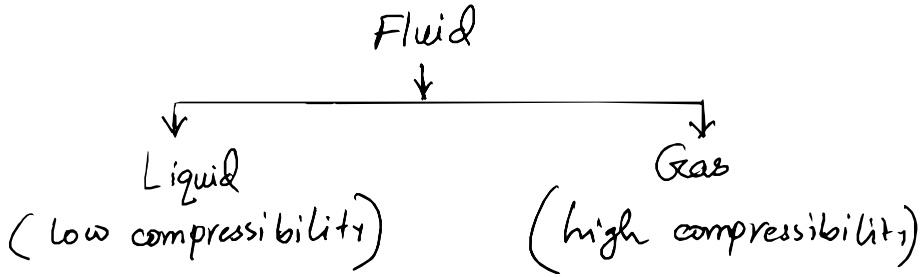
(I) $\frac{dP}{dx}$ (II) $\frac{d^2v}{dy^2}$

(III) $\frac{dm}{dt}$ (IV) $\frac{dv}{dt}$

P - Pressure, v - velocity

x, y - spacial location

m - mass, t - time



* <u>Ideal gas</u>: A gas is called ideal gas if it follows the expression bellow

$$\boxed{P = \rho R T}$$

Pressure (Pa) (N/m^2) (J/m^3)

→ Temperature (K)

→ Gas constant (J/kg K)

→ Density (kg/m^3)

* How do you define density?

* What does unit of P in J/m^3 mean??
* What type of energy is stored in a pressurised gas?
* How to obtain "R":

a) R is a gas specific constant. It's value varies from gas to gas.

b) There is "something" called "universal gas constant" that does not vary gas to gas.

$$R^* = 8.314 \text{ J/mol K}$$

* Remmember: $R = R^*/M$ → Molar mass (kg/mol)

R^* → Universal gas const. (8.314 J/mol K)

R → Specific gas const. (J/kg K)

* Determine the gas constant for

ⓐ H_2 ($M_{H_2} = 2$ g/mol)

ⓑ O_2 ($M_{O_2} = 32$ g/mol)

ⓒ air ($M_{air} = 29$ g/mol)

* The Ideal gas law can be re-rewriten as

$$P = \left(\frac{mRT}{V}\right)$$

* what will happen when

ⓐ V is decreased keeping all parameters same except pressure.

ⓑ V is decreased keeping all parameters same except T.

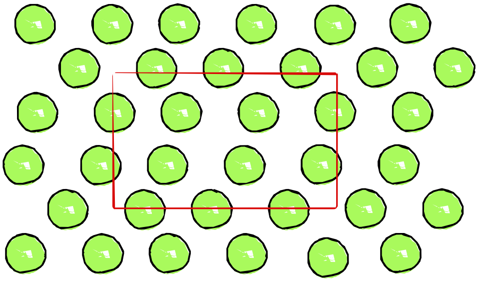
ⓒ T is changed without changing everything else except P.

Fluid as continuum

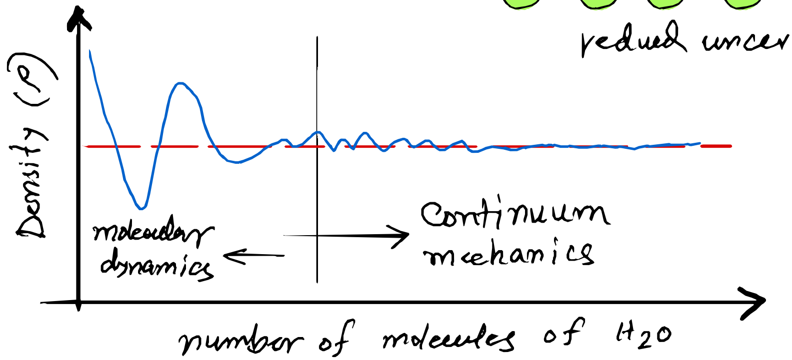
* Read Section 1.5 from white

→ To define micro-scale parameters (like ρ) we need a minimum number of atoms/molecules.

Lots of uncertainty on volume selection



reduced uncertainty



* Continuum is defined as an "hypothetical contineous media" that has identical fluid properties in micro-scale (~10^{-6} m)

* What about nano-scale (~10^{-9} m)?

→ What is the radius of atoms? (0.5~3.5 A°) for most elements

$$1A^{\circ} = 10^{-10}\, m = 0.1\, nm.$$

* Determine number of atoms in 1 mm^3 of Hellium gas (He) at room condition.

$$PV = nRT \qquad \text{(mol)}$$

$$\Rightarrow \quad 101325 \times 10^{-9} = n \times 8.314 \times 300$$

$$\Rightarrow \quad n = 4.06 \times 10^{-8} \text{ mol}$$

$$\therefore \quad N_{atom} = n N_A = 4.06 \times 10^{-8} \times 6.023 \times 10^{23}$$

$$\Rightarrow N_{atom} \approx 2.4 \times 10^{16} \approx 2.4 \times 10^{7} \text{ billion}$$

$\approx$ 24 million of billions

only in 1 mm^3 ??

* Determine number of atoms in 1 nm^3 of Helliumgas (He) at room condition.

$$n = \left(\frac{101325 \times 10^{-27}}{8.314 \times 300}\right) = 4.06 \times 10^{-26} \text{ mol}$$

$$\therefore N_{atom} = 4.06 \times 10^{-26} \times 6.023 \times 10^{23}$$

≈ 0.024 (not even a single atom!!)

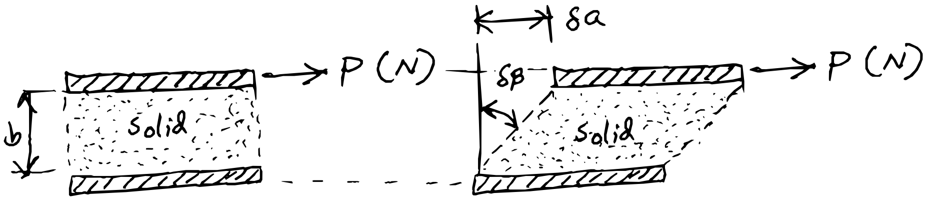
* Do it for 1 um^3 of He.

$$N_{atom} \approx 24 \times 10^{6}$$ (24 millions)

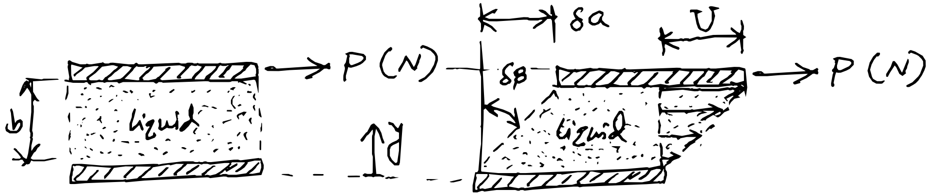
"What is the diameter of human hair?"

Viscosity

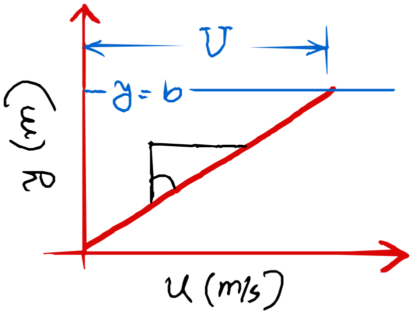
* It is a fluid property.
* Consider an hypothetical experiment as:



* Applied force P is balanced by the shear stress developed in the material. $P = \tau A$
* Now replace the "solid" material between two plates by a liquid (Fluid)



* Starts to flow (continuous deformation)
* Velocity becomes $u(y) = U(y/b) \longrightarrow$ (linear)
* Why the velocity profile is linear?? (Guess)



* From figure,

$$\frac{du}{dy} = \left(\frac{U}{b}\right)$$

(linear)

* * In small time increment: $\delta\beta \approx \tan(\delta\beta) = \left(\frac{\delta a}{b}\right)$

* Again $\delta a = U\delta t$ (displacement = velocity × time)

Thus, $\delta\beta = \left(\frac{U\delta t}{b}\right) \longrightarrow \delta\beta = f(U, t)$

↳ Depends on P.

* Now we see the time dependence!! So we introduce strain rate (Not strain, as we did for solid) as:

$$\dot{\gamma} = \lim_{\delta t \to 0}\left(\frac{\delta\beta}{\delta t}\right) = \left(\frac{U}{b}\right) = \left(\frac{du}{dy}\right)$$

* Since $\dot{\gamma}$ increase with P (or similarly $\tau = P/A$)

$$\tau \propto \dot{\gamma}$$

$$\Rightarrow \tau = \mu\left(\frac{du}{dy}\right)$$ (Newton's law of viscosity)

* Fluids that follows this equation are called "Newtonian fluids"

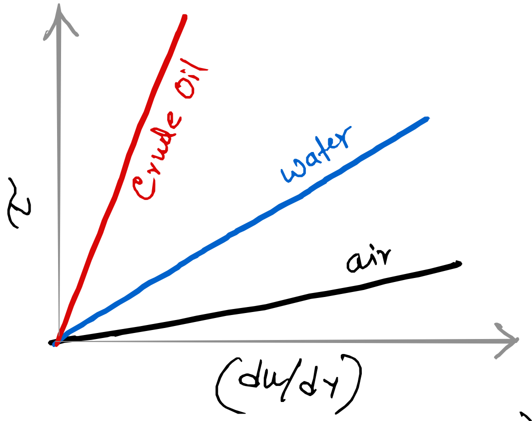
$$\boxed{\tau = \mu\left(\frac{du}{dy}\right)}$$

→ rate of deformation (effect)

↓ Force (Cause)

→ Fluid Property (Viscosity)

* Newtons law of viscosity: $\tau = \mu \left(\frac{du}{dy}\right)$

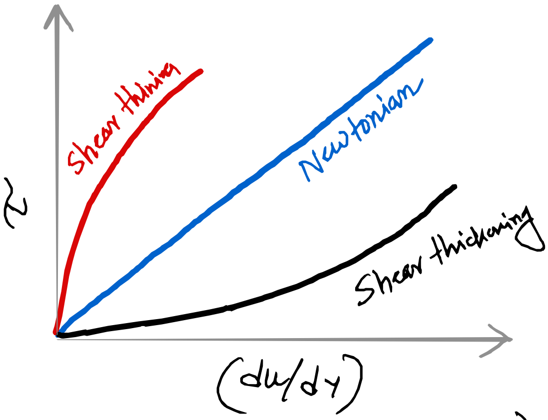


(See fig 1.6 from textbook)

Newtonian Vs Non-Newtonian fluids

For non-Newtonian fluid, $\tau = \mu_{ap}\left(\frac{du}{dy}\right)$

$\uparrow$ (Not constant!!)



(See fig 1.7 from textbook)

Dynamic Vs Kinematic Viscosity

- $\tau = \mu \left(\frac{du}{dy} \right)$ "what is the unit of μ ?"
- Density gives a sense of heaviness !!
- Viscosity μ gives a sense of what ??

(See video on youtube Video-1)

Ans → { Resistance to flow.
Resistance to momentum diffusion

- Does density helps to resist flow ?? (Think !!)

→ * Can a low density fluid have higher viscosity ? or visc-versa ?

- What about a new parameter that gives density specific viscosity :

$$\nu = \frac{\mu}{\rho} \qquad (\rho = \text{density})$$

Here, μ = Dynamic viscosity (Pa·s)
(Absolute viscosity)

ν = Kinematic viscosity (m^2/s)

Interesting !!

Temperature dependence of Viscosity

* What is the source of viscosity ?

→ Internal friction (Collision of molecules during flow)

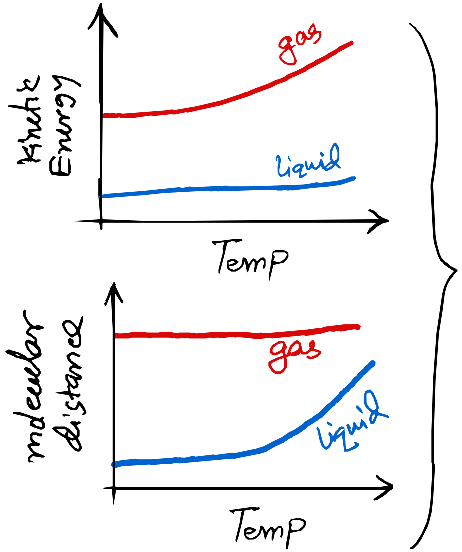
* Increasing temperature

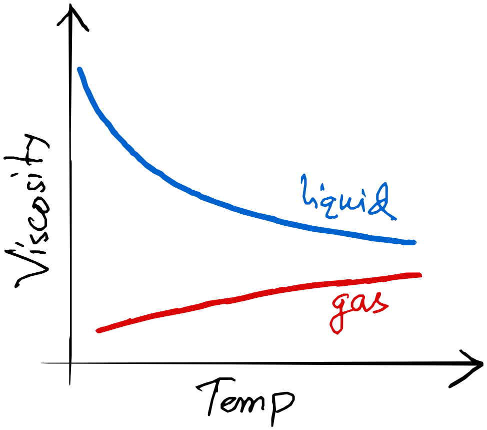
ⓐ Increases kinetic energy of molecules
(higher internal collision)

ⓑ Increases distance between molecules
(lower internal collision)

* For gas → effect of change in molecular distance is negligible

* For liquid → Increase in kinetic energy is negligible





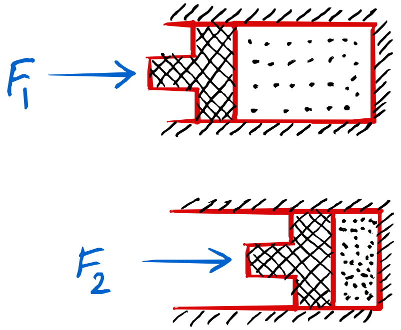
(See fig 1.8 of the text book)

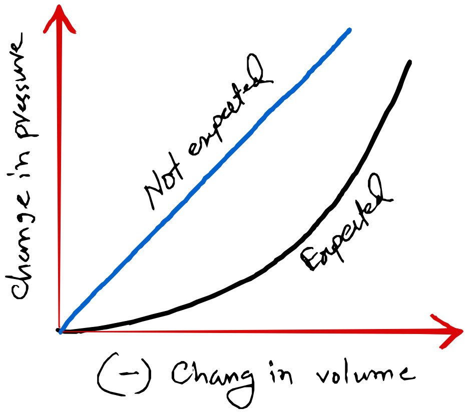
Compressibility

- * Can pressure change the volume?
  → How about squeezing an air filled baloon?
- * What could be a good cause and effect relation between change in pressure and change in volume? (constitutive relation)
- * First guess: $dp \propto d\forall$
  → does increase in P causes increase in $\forall$?

$\forall$ = volume
P = pressure

- * Second guess: $dp \propto -d\forall$
  → does same pressure change causes same amount of volume change for any initial volume?





- * More and more pressure is required to decrease the volume as the volume approaches to zero.

* Third guess: $dp \propto -\left(\frac{d\forall}{\forall}\right)$

→ Does it make sense?? why?

what happens when $\forall \rightarrow 0$?

* Introduce a proportionality constant as

$$dp = -E_v\left(\frac{d\forall}{\forall}\right)$$

$$\Rightarrow E_v = -\left\{\frac{dp}{(d\forall/\forall)}\right\}$$

{Compressibility coefficient known as bulk modulus}

* Also can be written as

$$E_v = +\frac{dp}{(d\rho/\rho)}$$

(unit of E_v is same as pressure)

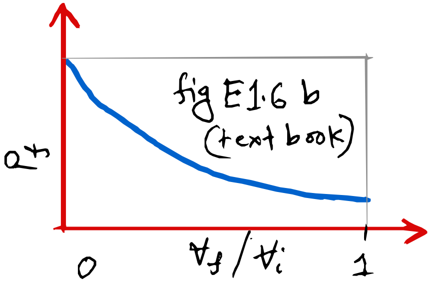
* High value of E_v means what?

→ E_v represents required change in pressure to change the volume (100 %). weird??

* Can we use the expression as

$$E_v = -\frac{P_f - P_i}{(\forall_f - \forall_i)/\forall_i}$$

Ans: No



Compression & expansion of Gas

* For ideal gas $P = \rho R T$

$R =$ → For isothermal process : $\frac{P}{\rho} = RT$ = constant

→ For isentropic process : $\frac{P}{\rho^k}$ = constant

* what is K ??

→ K is an constant (unitless) that describe the ratio of specific heat C_p and C_v

C_v → const volume

C_p → const pressure

$$\therefore \quad K = \left(\frac{C_p}{C_v}\right)$$

→ Also, $C_p - C_v = R$ } (Thermodynamics)

For air $K = 1.4$ (known)

* Since $E_v = +\left(\frac{dP}{d\rho/\rho}\right)$

→ For isothermal process $P = C_1 \rho \Rightarrow dP = C_1 d\rho$

$$\therefore E_v = C_1 \rho = P$$

→ For isentropic process $P = C_2 \rho^k \Rightarrow dP = C_2 k \rho^{k-1}$

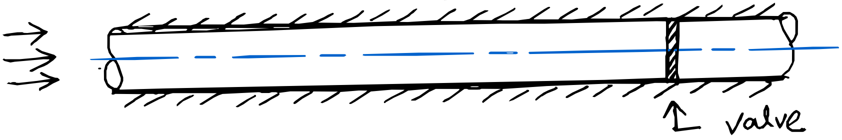
$$\therefore E_v = C_2 k \rho^k = kP$$

* Bulk modulus → Is it a purely fluid property?

→ why would it depend on the process?

Speed of sound

* Consider water flowing through a pipe.



* What happens when the valve is suddenly closed?

→ Finite time is required for the upstream to understand the valve closure.

→ How do the upstream sense (change of what physical parameter) the valve closure?

* Ans: pressure.

→ change in the pressure travels at a certain speed. what is sound??

→ Sound travels at same speed as the "pressure wave" travels. This speed is known as acoustic velocity (or speed of sound)

→ The speed of sound (will be shown later) is related to change in pressure and change in density as below:

$$c = \sqrt{\frac{dP}{d\rho}} \quad \text{or} \quad c = \sqrt{\left(\frac{E_v}{\rho}\right)}$$

→ For sudden change (isentropic process)

$E_v = kP$

* Using ideal gas law we get, $c = \sqrt{KRT}$.

* Derive the expression for c for iso-thermal process. (why $K=1$ works here). what does $K=1$ mean ($K = C_p/C_v$). when can $C_p = C_v$ be true?

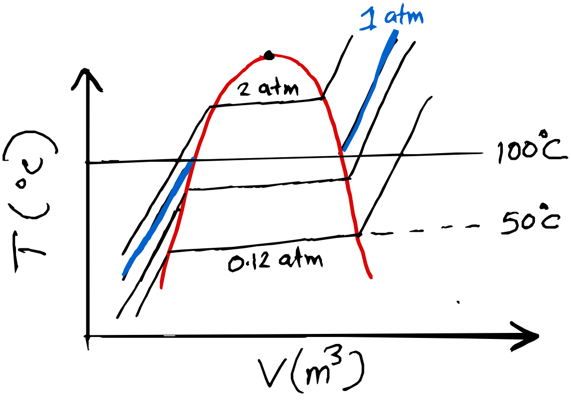
Vapor Pressure

How can we evaporate water?

* ⓐ Increasing temperature. (Pressure constant)
ⓑ Decreasing pressure. (Temperature constant)

* Remmember the T-V diagram from thermodynamics

How does a pressure pot work?

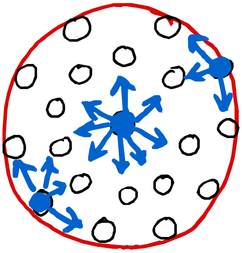


* lower vapor pressure causes evaporation at lower temp.

* May cause "cavitation" in turbo machinuries such as pumps/water turbines. (Later)

Surface tension

- Important for low volume and high surface cases. (Capilary flow, water droplet etc).
- Due to imbalanced cohesive force the liquid (rarely gas) "skin" or membrane is subjected to a tensile stress.



- The surface always tries to reduce its size.
- More surface requires more energy.
- Droplets are spherical & they tries to merge together. (Coalescence, video - 2)

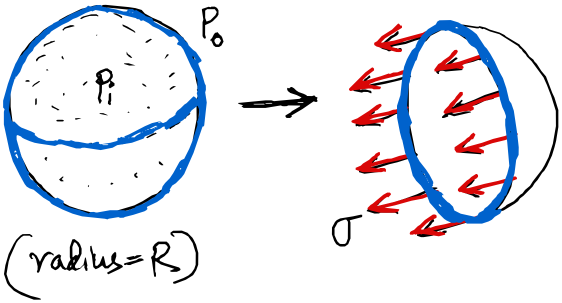
- The tensile stress acts on the plane of the surface and act on a line.
- Surface tension is thus defined as

  "Intensity of molecular attraction per unit length along any line"

- Surface tension has a unit of N/m and dimension FL^{-1} (in FLT system).

Pressure inside a droplet

- Consider a water droplet of radius R. The inside and outside pressures are P_i and P_o respectively.



- The pressure difference $P_i - P_o$ is balanced by the surface tension force.

Pressure force = Surface tension force

$$\Rightarrow (P_i - P_o)\cdot \underbrace{\pi R^2}_{\text{(area on which pressure works)}} = \sigma \cdot \underbrace{2\pi R}_{\text{(line on which surface tension works)}}$$

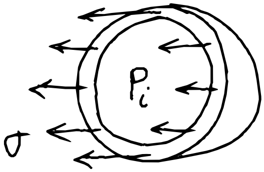
$$\Rightarrow P_i = P_o + \left(\frac{2\sigma}{R}\right) \qquad \boxed{\Delta P = \left(\frac{2\sigma}{R}\right)}$$

- Smaller droplets of same liquid has more inside pressure.

- What happens when $\sigma = 0$. How the liquid is expected to behave?

* Pressure inside bubble

f



Pressure force = Surface tension force

$$\Rightarrow (P_i - P_o)\pi R^2 = \sigma\cdot(2\pi R) \times 2$$

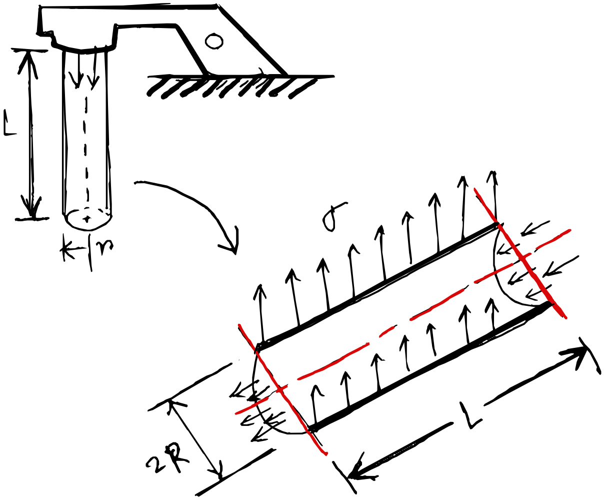
↑ why?

$$\Rightarrow \boxed{\Delta P = \frac{4\sigma}{R}}$$

* Pressure inside jet

Pressure force

= surface tension force



$$\Rightarrow \Delta P(2RL) = \sigma\, 2L$$

$$\Rightarrow \boxed{\Delta P = \frac{\sigma}{R}}$$

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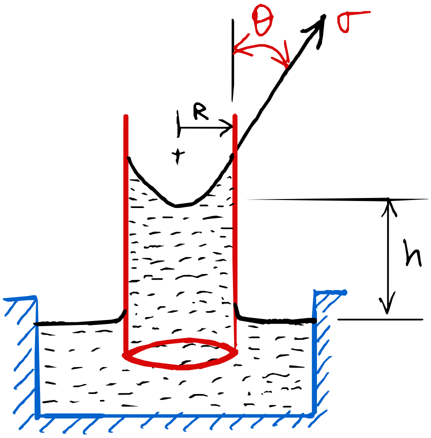
* Notice that

$$\boxed{\Delta P \sim \frac{\sigma}{R}} \longrightarrow \begin{cases} \text{Drop, } \Delta P = 2\sigma/R \\ \text{Bubble, } \Delta P = 4\sigma/R \\ \text{Jet, } \Delta P = \sigma/R \end{cases}$$

Capilary tube

- * Why fluid surfaces moves up's/down's when an inverted tube is submerged.



- * The surface tension force (N) must support the weight (N) of the liquid column.

$$\begin{pmatrix}\text{Weight of}\\ \text{liquid}\\ \text{column}\end{pmatrix} = \begin{pmatrix}\text{Surface}\\ \text{tension}\\ \text{force}\end{pmatrix}$$

$$\Rightarrow (\rho g)(\pi R^2 h) = \sigma \cdot (2\pi R) \cos\theta$$

$$\Rightarrow h = \left(\frac{2\sigma\cos\theta}{\rho g R}\right)$$

{θ is known as contact angle}

- * The angle θ is a measure of solid surface's "wettability" to the liquid (remmember θ is specific to solid-liquid pair)

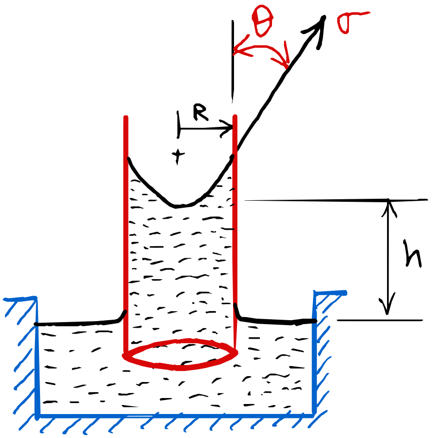
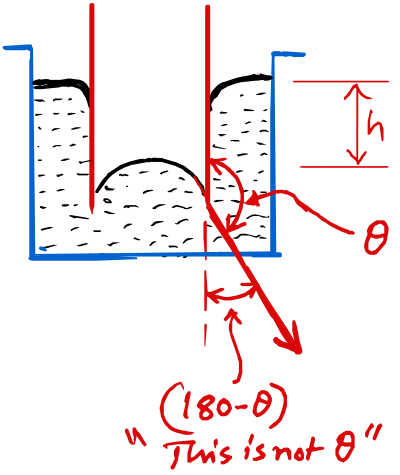
- * For solid-water pair:
  - → if $\theta < 90$ the pair is called "hydrophilic"
  - → if $\theta > 90°$ the pair is called "hydrophobic"

* What happens when $\theta > 90°$?

$$h = \left(\frac{2\sigma\cos\theta}{\rho g R}\right) < 0$$

$\boxed{\text{* Thus } h \text{ is } \ominus \text{ve}}$

* For hydrophobic pair (glass-mercury) the liquid level drops inside capilary tube.

Hydrophilic (wettable)	Hydrophobic (unwettable)
 Glass-water pair	 Glass-Mercury pair.

* Such rise/drop of liquid level is called "capilary action."

[*] Remmember:

* water-glass : $\theta \approx 0°$

mercury-glass : $\theta \approx 130°$

[*] Other things to remmember:

a) $\sigma_{H_2O} \approx 7.2 \times 10^{-2}$ N/m (@ room temp)

b) Standard temperature & pressure (STP)

0°c and 1 atm

* c) Room temperature & pressure (RTP)

25°c and 1 atm

* d) Density of water @ 4°c $\approx$ 1000 kg/m^3

* e) Universal gas constant, $R^* = 8.314$ J/kgk

* f) Molar weight of air, $M_{air} = 29$ g/mol.

g) Density of air (@ RTP), $\rho_{air} = 1.2$ kg/m^3

* h) 1 atm $\cong$ 101325 Pa = 101.325 kPa

i) 1 atm $\cong$ 14.7 psi

* j) $g = 9.8$ m/s^2 (unless mensioned)

k) $K_{air} = 1.41$ (isentropic coefficient of air)

* → highly expected to memorise !!!