ME 247
(Engineering Mechanics-Statics)

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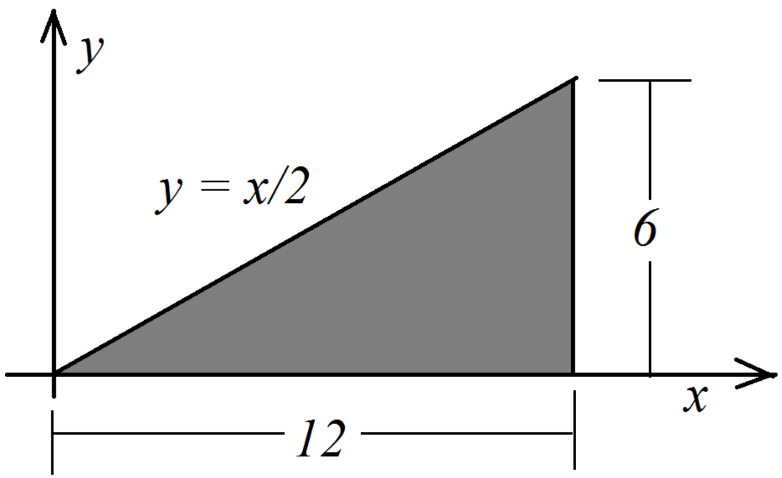
Center of Gravity of an Area (CG) (Centriod)

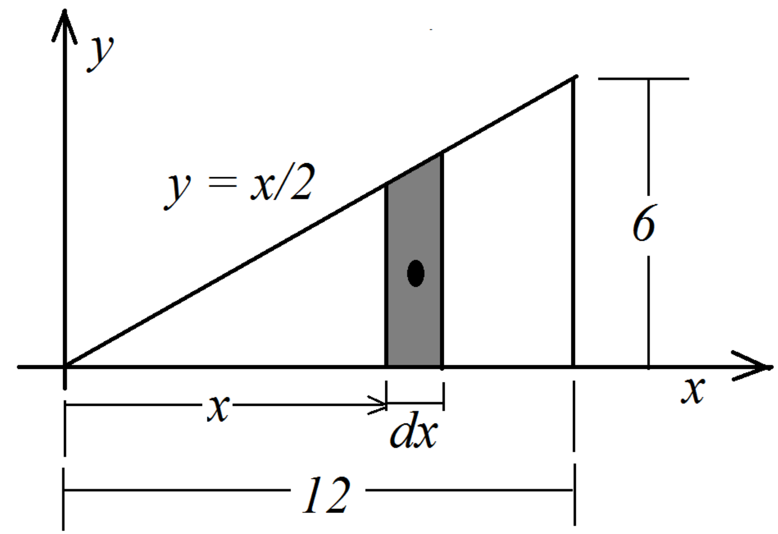
$$\bar{x}A = \int \acute{x}\, dA \qquad \bar{y}A = \int \acute{y}\, dA$$

** Here $\acute{x}$ *and* $\acute{y}$ are the centroid of the selected differential are ‘*dA*’.

Problems (Center of Gravity)

Example-1: Find the location of centroid for the triangular area shown.





$$A = \int dA = \int_0^{12} y\,dx \qquad [y = f(x) = x/2]$$

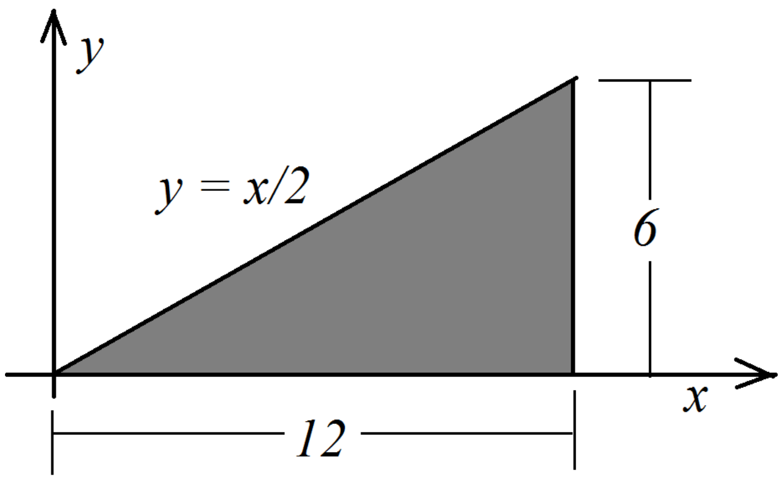
$$\bar{x}A = \int \acute{x}\,dA = \int_0^{12} x\,y\,dx \qquad [\acute{x} = x]$$

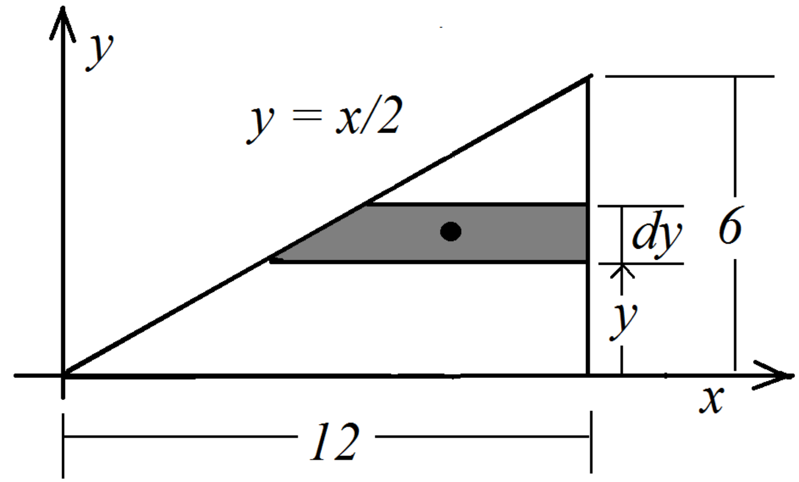
$$\bar{y}A = \int \acute{y}\,dA = \int_0^{12} \frac{y}{2}\,y\,dx \qquad [\acute{y} = y/2]$$

Answer: $\bar{x} = 8$ (unit)
$\bar{y} = 2$ (unit)

Problems (Center of Gravity)

Example-1: Find the location of centroid for the triangular area shown.





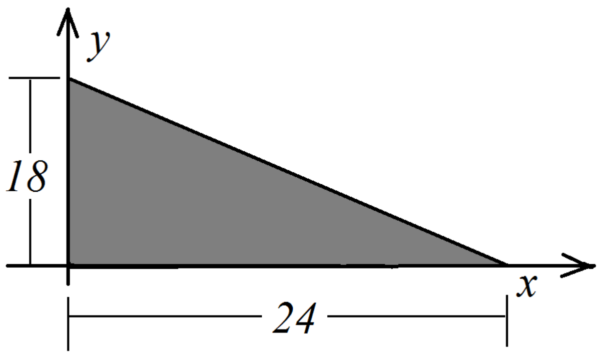
$$A = \int dA = \int_0^6 (12 - x)dy \qquad [x = f(y) = 2y]$$

$$\bar{x}A = \int \acute{x}dA = \int_0^6 \left(x + \frac{12 - x}{2}\right)(12 - x)dy \qquad \left[\acute{x} = \left(x + \frac{12 - x}{2}\right)\right]$$

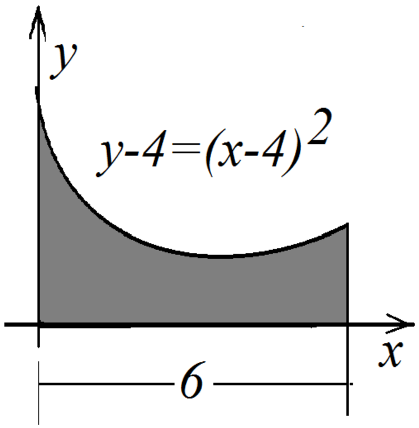
$$\bar{y}A = \int \acute{y}dA = \int_0^6 y\,(12 - x)\,dy \qquad [\acute{y} = y]$$

Problems (Center of Gravity)

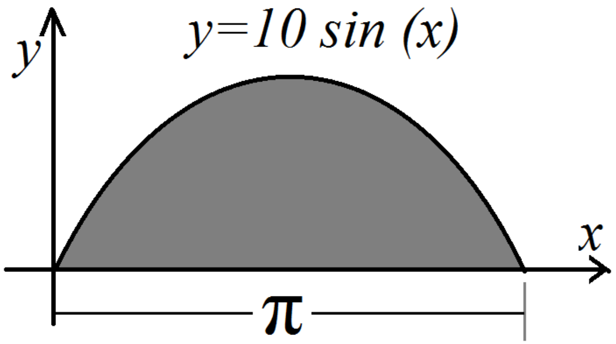
Assignment-1: Find the location of centroid for the areas shown. (Use both axis as reference axis)



(a)



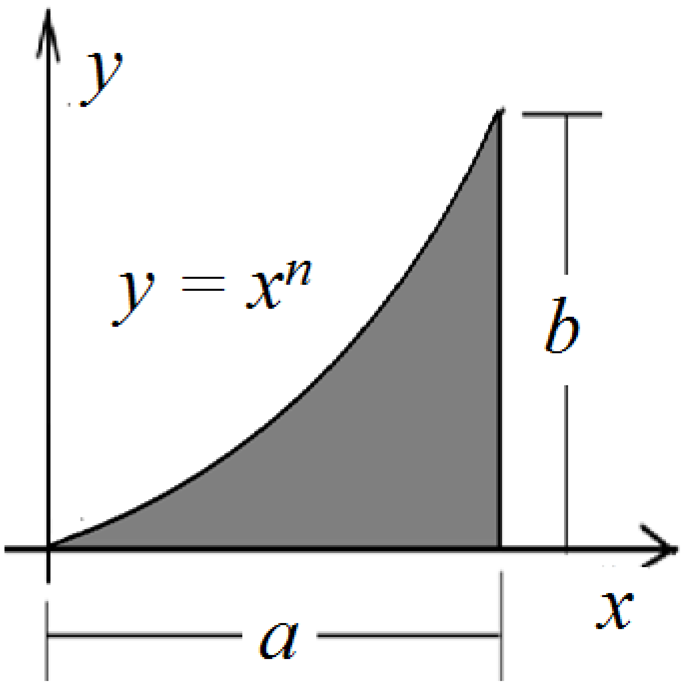
(b)



(c)

Problems (Center of Gravity)

Example-2: Find the location of centroid for the area shown.



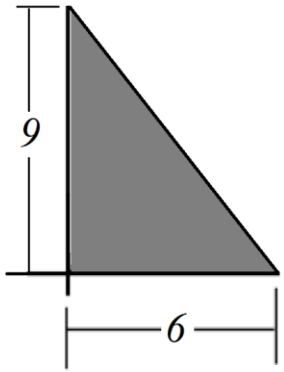
$$A = \int dA = \int_0^a x^n\, dx = \frac{ab}{n+1}$$

$$\bar{x}A = \int \acute{x}\, dA = \int_0^a x \times x^n dx = \int_0^a x^{n+1} dx$$

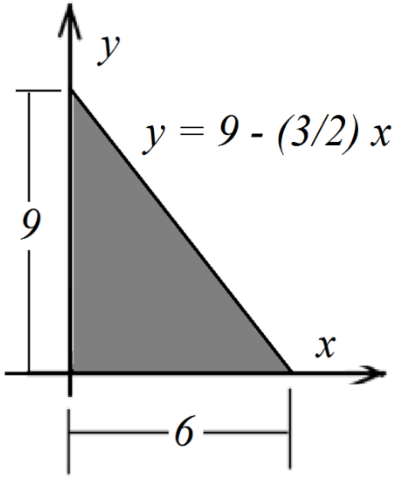
$$\bar{y}A = \int \acute{y}\, dA = \int_0^a \frac{y}{2}\, y\, dx = \frac{1}{2}\int_0^a x^{2n} dx$$

Problems (Center of Gravity)

Example-3: Find the location of centroid for the triangular area shown.



$$A = \int dA = \int_0^6 (9 - \frac{3}{2}x)\, dx$$

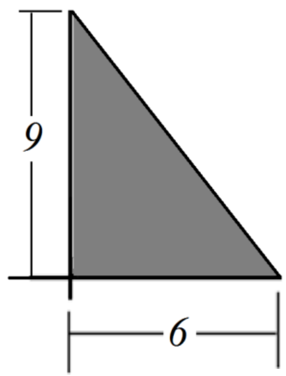


$$\bar{x}A = \int \acute{x} dA = \int_0^6 x\left(9 - \frac{3}{2}x\right) dx$$

$$\bar{y}A = \int \acute{y} dA = \int_0^6 \frac{y}{2}\, y\, dx = \frac{1}{2}\int_0^6 \left(9 - \frac{3}{2}x\right)^2 dx$$

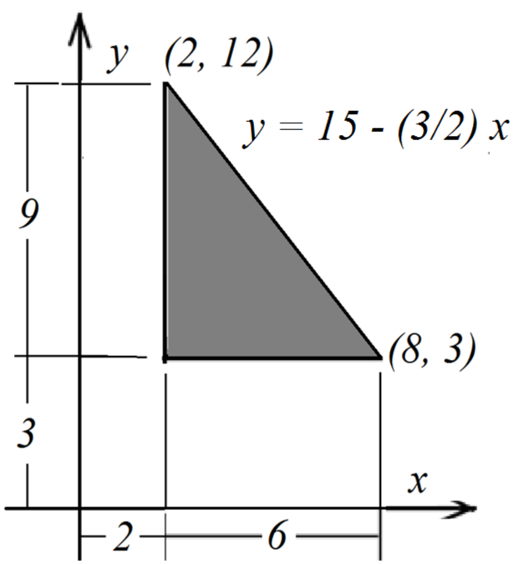
Problems (Center of Gravity)

Example-3: Find the location of centroid for the triangular area shown.



$$A = \int dA = \int_2^8 (y-3)\,dx = \int_2^8 \left[\left(15 - \frac{3}{2}x\right) - 3\right] dx$$

$$\bar{x}A = \int \acute{x}\,dA = \int_2^8 x\left[\left(15 - \frac{3}{2}x\right) - 3\right] dx$$

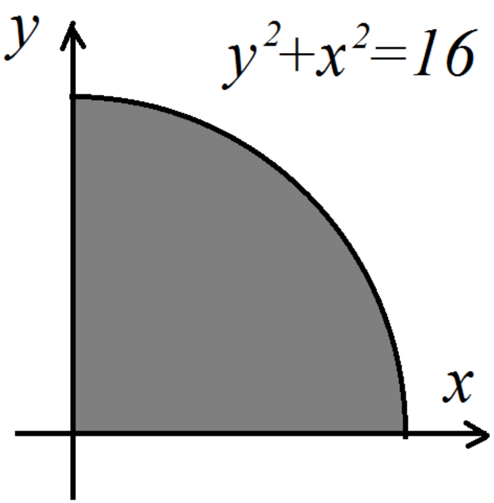


$$\bar{y}A = \int \acute{y}\,dA = \int_2^8 \left(\frac{y-3}{2} + 3\right)(y-3)\,dx$$

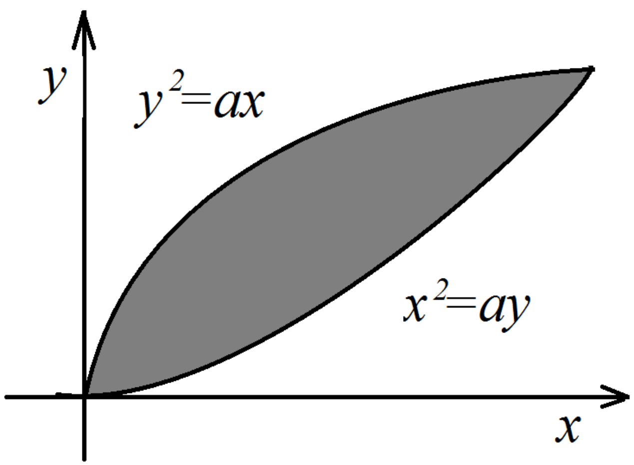
$$= \frac{1}{2}\int_2^8 \left[\left(15 - \frac{3}{2}x\right)^2 - 9\right] dx$$

Problems (Center of Gravity)

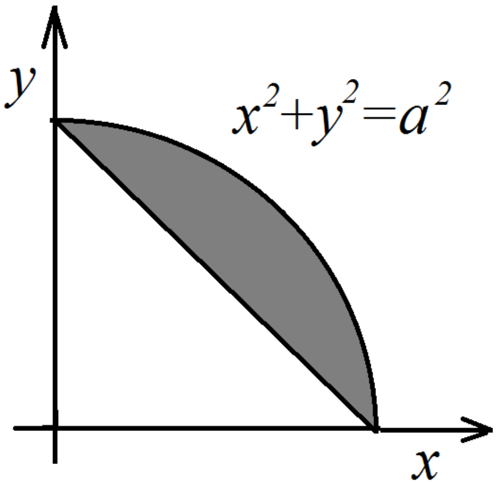
Example-4, 5: Find the location of centroid for the area shown.



Example – 4



Example - 5(a)



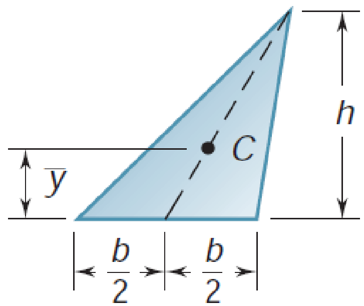
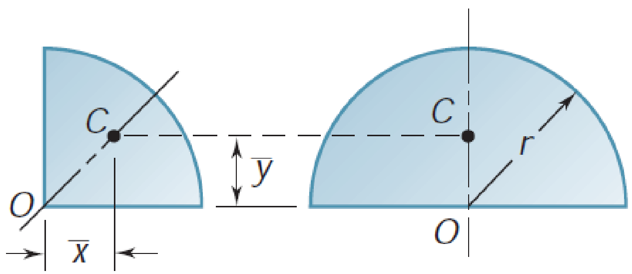
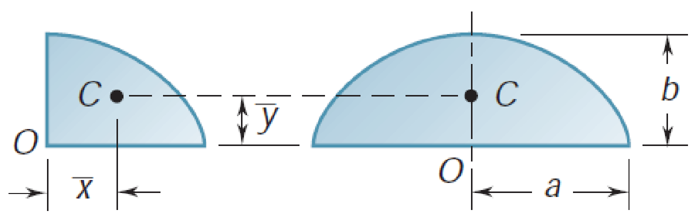
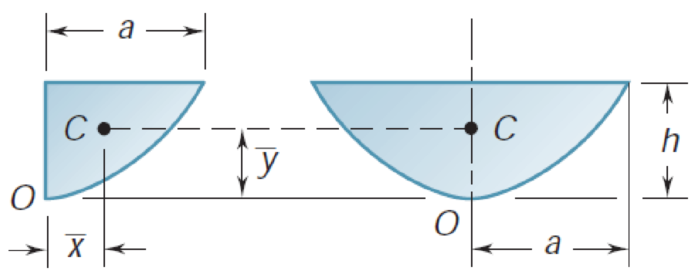
Example – 5(b)

Center of Gravity of a Compound Area

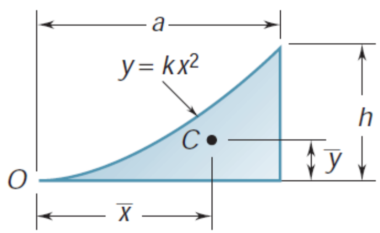
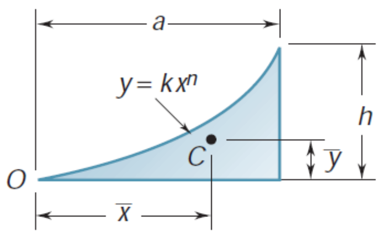
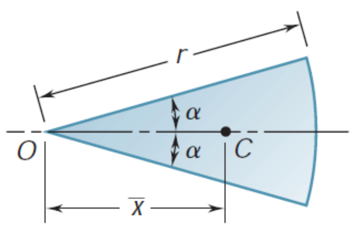
$$\bar{x} = \frac{\sum \bar{x}_i A_i}{\sum A_i} \qquad \bar{y} = \frac{\sum \bar{y}_i A_i}{\sum A_i}$$

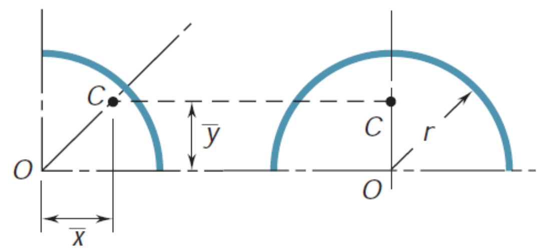
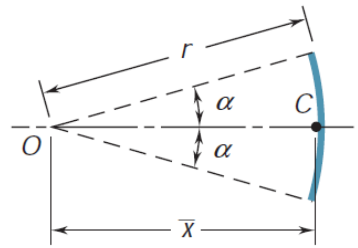
- The required area is created by adding/subtracting some simple areas A_1, A_2 ..... A_n
- Then the centroids of individual simple areas are calculated.
- Finally above two equations are used to calculate the centroid of desired compound area.

Center of Gravity

	$\bar{x}$	$\bar{y}$	Area
		$\frac{h}{3}$	$\frac{bh}{2}$
	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
	0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
	$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
	0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
	$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
	0	$\frac{3h}{5}$	$\frac{4ah}{3}$

Center of Gravity

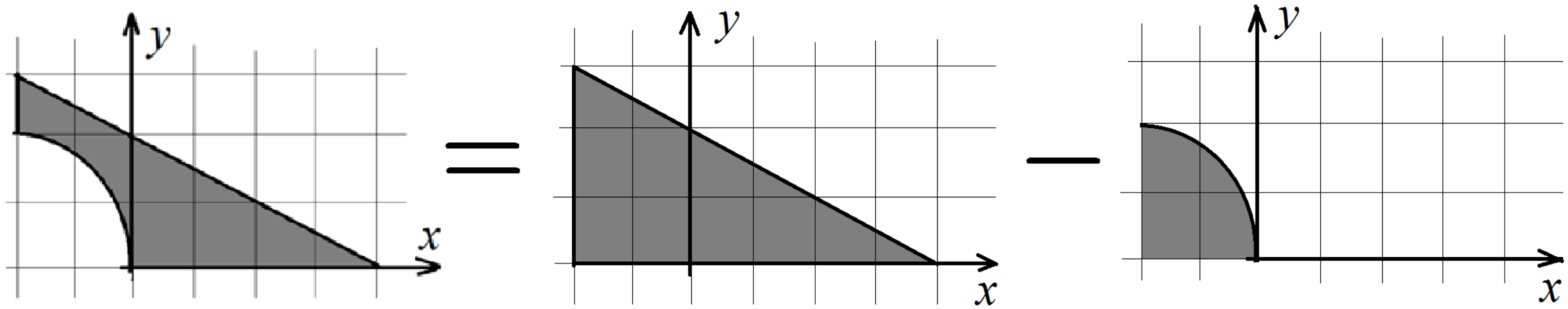
	$\bar{x}$	$\bar{y}$	Area
	$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
	$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
	$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2

	$\bar{x}$	$\bar{y}$	Area
	$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
	0	$\frac{2r}{\pi}$	πr
	$\frac{r\sin\alpha}{\alpha}$	0	$2\alpha r$

Memorize the location of CG for all the area/line mentioned above.

Problems (Center of Gravity)

Example-6: Find the location of centroid for the triangular area shown. Both in x and y direction small segment represents 10 mm.



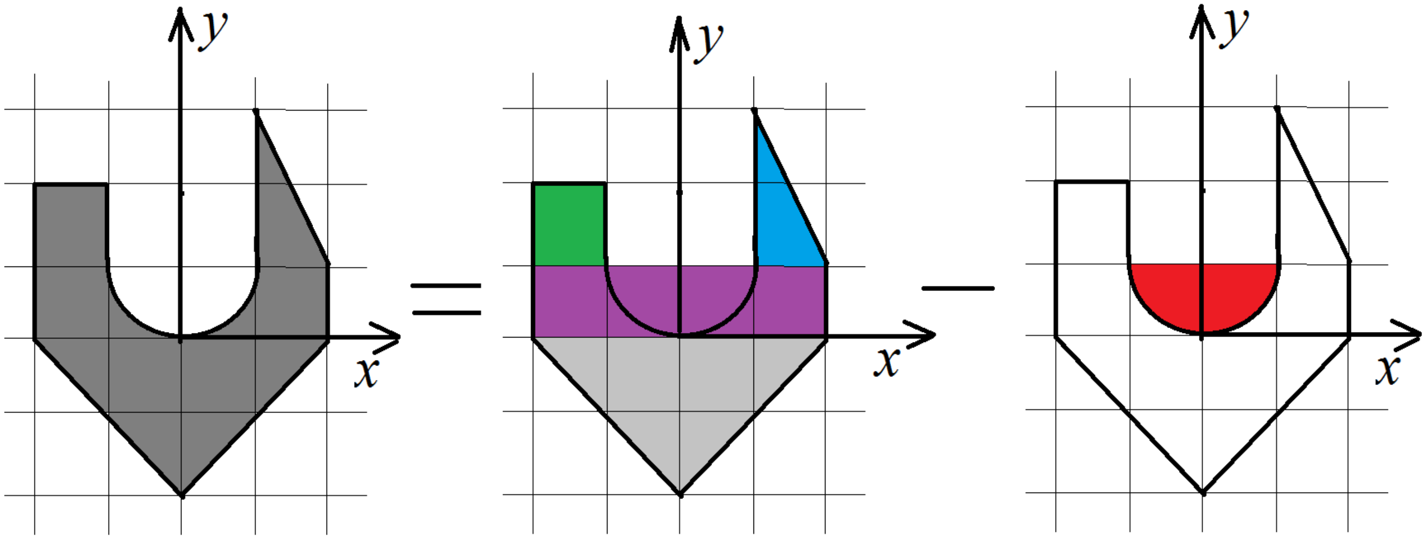
Component	Area, A (unit)^2	$\bar{x}$ (unit)	$\bar{y}$ (unit)	$\bar{x}A$ (unit)^3	$\bar{y}A$ (unit)^3
Triangle	(½)*60*30 = 900	0	+ 10	0	+ 9000
Quarter Circle	-(π/4)*400 = -314.1	- 11.51	+ 8.49	+ 3615.3	- 2666.7
$\sum Sum$	+ 585.9			+ 3615.3	+ 6333.3

$$\bar{x} = \frac{\sum \bar{x}A}{\sum A} = \frac{3615.3}{585.9} = 6.17 \quad (unit)$$

$$\bar{y} = \frac{\sum \bar{y}A}{\sum A} = \frac{6333.3}{585.9} = 10.8 \quad (unit)$$

Problems (Center of Gravity)

Example-7: Find the location of centroid for the area shown.



Along both axis Small Segment represents 10 unit

Component	Area, A $(unit)^2$	$\bar{x}$ (unit)	$\bar{y}$ (unit)	$\bar{x}A$ $(unit)^3$	$\bar{y}A$ $(unit)^3$
Rectangle	+ 40*10 = + 400	0	+ 5	0	+ 2000
Semi-Circle	$-(1/2)*\pi*10^2$ = -157.1	0	+5.76	0	-904.8
Rectangle	+10*10= +100	-15	+15	-1500	+1500
Triangle	+(1/2)*10*20 = +100	+13.3	+16.7	+1333	+1667
Triangle	+ (1/2)*40*20 = +400	0	-6.67	0	-2668
Sum	+842.9			-167	+1594.2

$$\bar{x} = \frac{\sum \bar{x}A}{\sum A}$$

$$= 0.2 \quad (unit)$$

$$\bar{y} = \frac{\sum \bar{y}A}{\sum A}$$

$$= 1.9 \quad (unit)$$

Always Remember

- The sign of the areas are not based on their location. It is based on whether they are being added or subtracted.
- The sign of the coordinate of the elemental area are based on their location. It is not based on whether they are being added or subtracted.

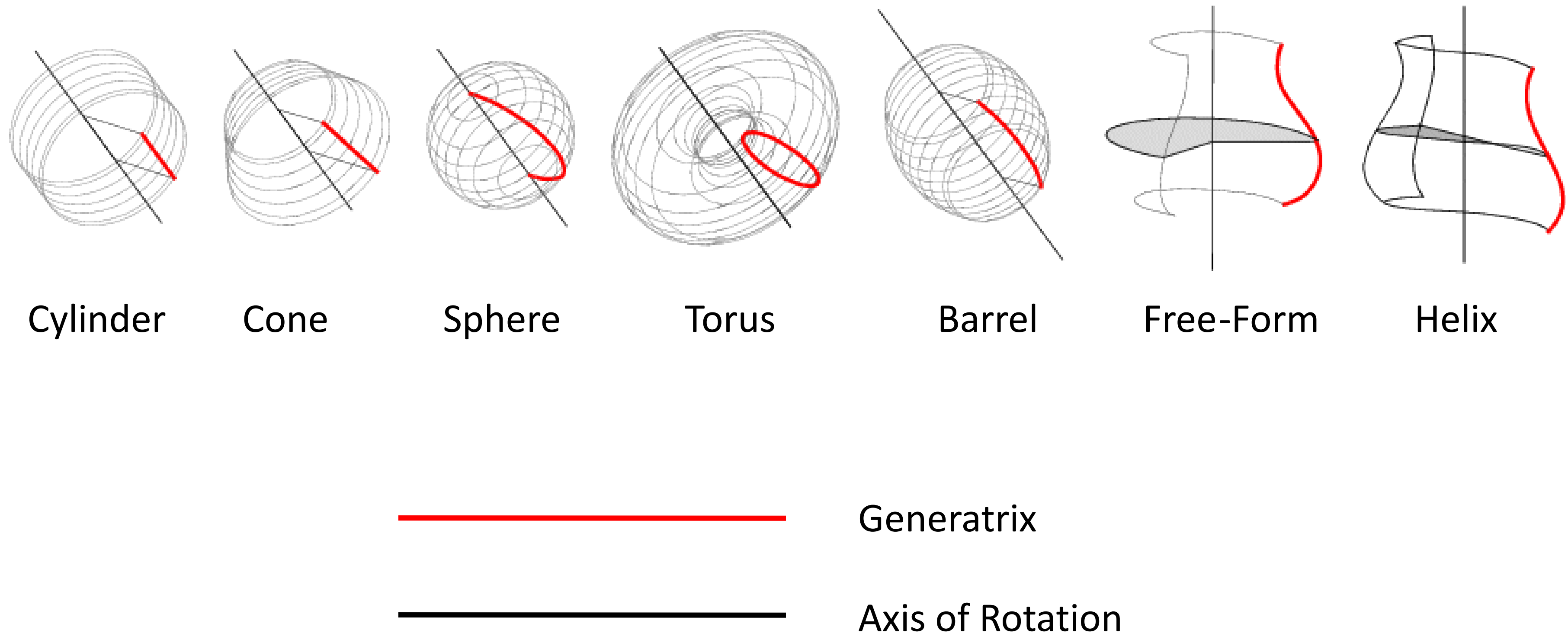
Pappus-Guldinus Theorem

- The are of a surface of revolution is equal to the length of the generating curve times the distance traveled by the centroid of the curve while the area is being generated.

$$A = L * \delta$$

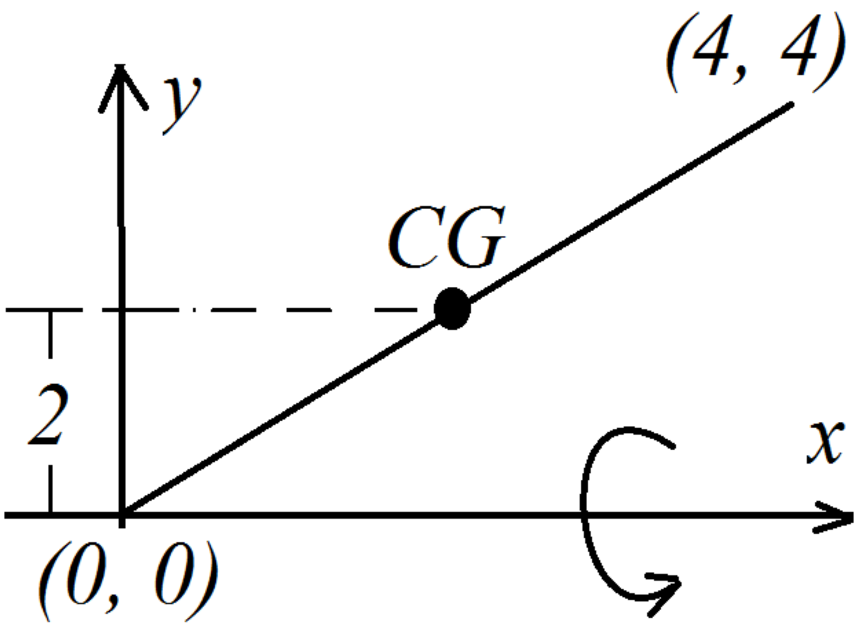
Pappus-Guldinus Theorem

$$A = L * \delta$$



Problems (Center of Gravity)

Example-8: Determine the surface area if the line passing through (0, 0) and (4, 4) is revolved 360^0 with respect to *x*- axis.



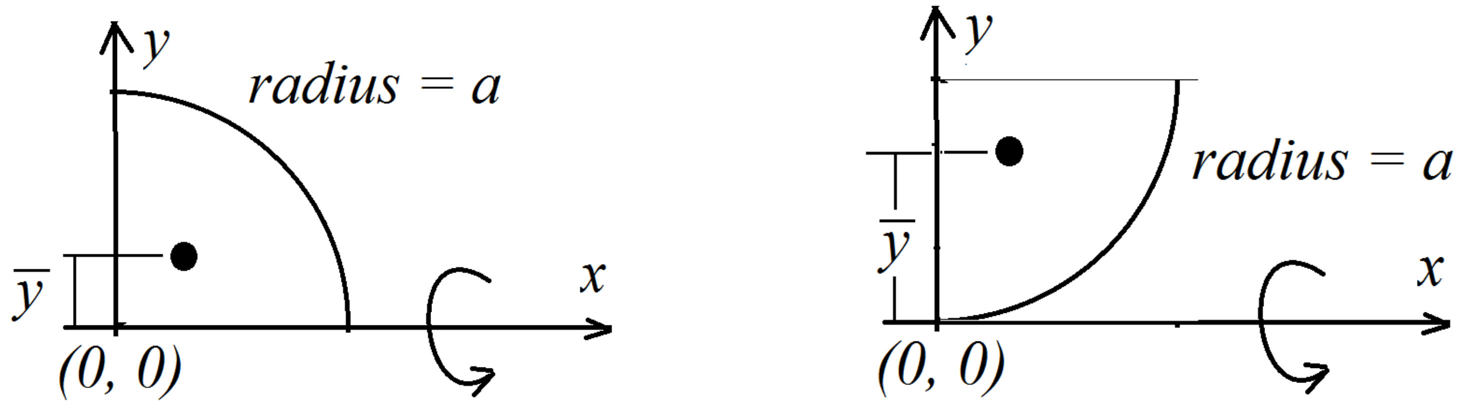
Area, $A = L*\delta$

$= [sqrt(4^2+4^2)]*[2*\pi*2]$

$= 71.09\ (unit)^2$

Problems (Center of Gravity)

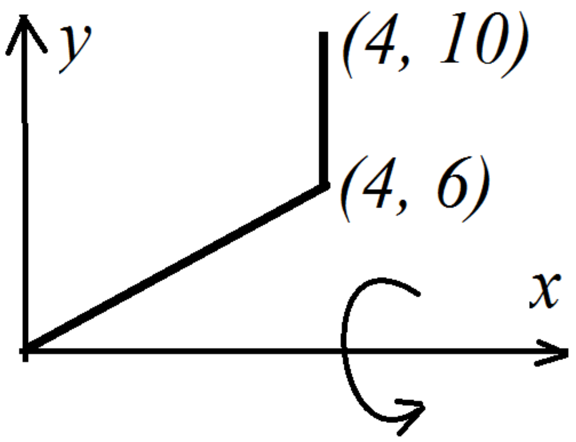
Example-9: Determine the surface area of the surface of revolution. (180^0 revolution about x-axis)



(a) (b)

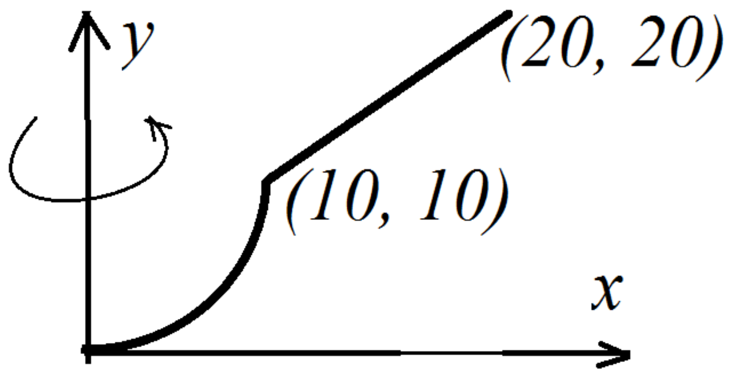
Problems (Center of Gravity)

Assignment-2: Determine the surface area of the surface of revolution.



180^0 revolution about x-axis

(a)



120^0 revolution about y-axis

(b)

Centroid of a Volume

$$\bar{x} = \frac{\sum \bar{x}_i W_i}{\sum W_i} \qquad \bar{y} = \frac{\sum \bar{y}_i W_i}{\sum W_i} \qquad \bar{z} = \frac{\sum \bar{z}_i W_i}{\sum W_i}$$

If the object is of homogenous material (ρ=constant) than $W \alpha V$.

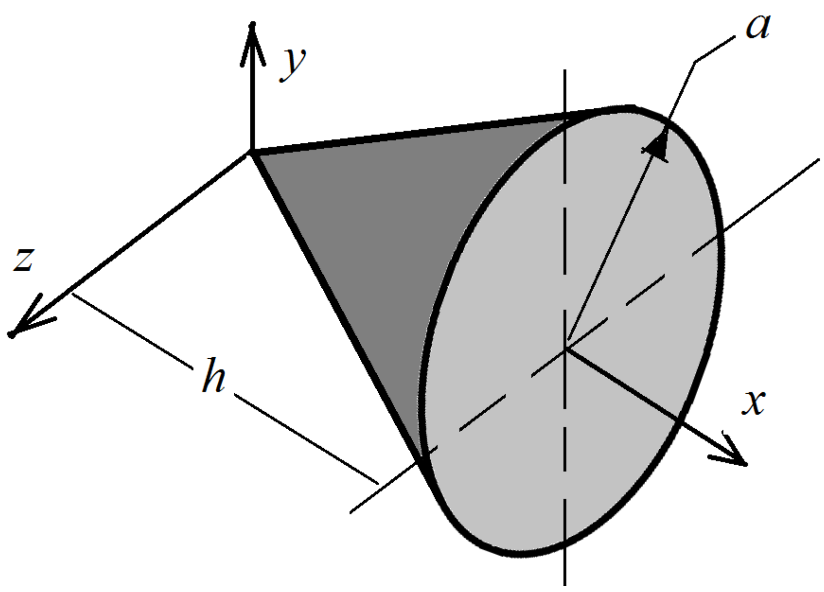
$$\bar{x} = \frac{\sum \bar{x}_i V_i}{\sum V_i} \qquad \bar{y} = \frac{\sum \bar{y}_i V_i}{\sum V_i} \qquad \bar{z} = \frac{\sum \bar{z}_i V_i}{\sum V_i}$$

Above equations are very convenient to use but in doing so need to know the centroids of simple volumes which can be found by using following equations:

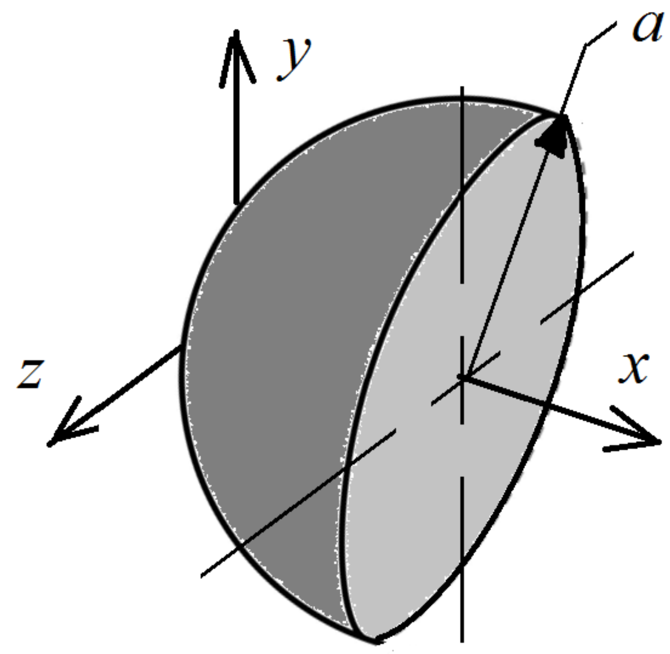
$$\bar{x}V = \int \acute{x} dV \qquad \bar{y}V = \int \acute{y} dV \qquad \bar{z}V = \int \acute{z} dV$$

Problems (Center of Gravity)

Example-10: Determine the centroids for the volumes shown.



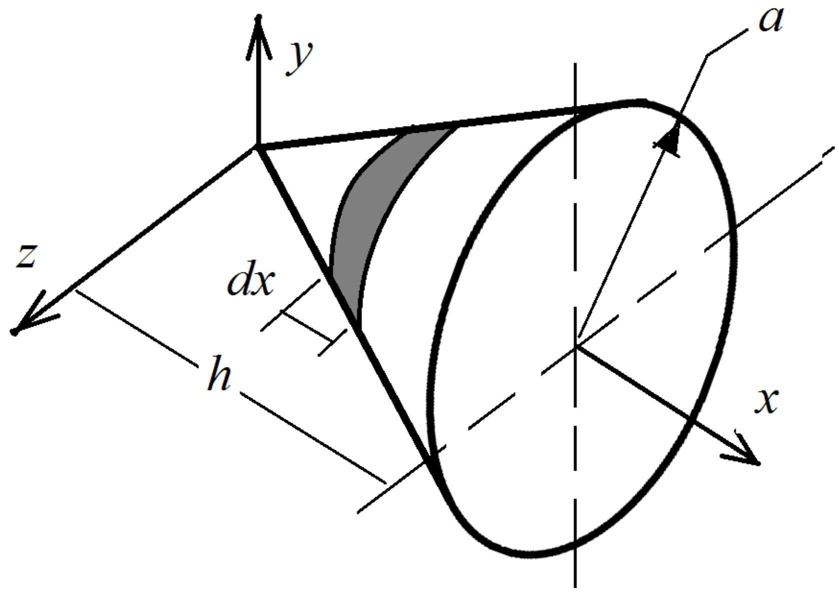
(a)

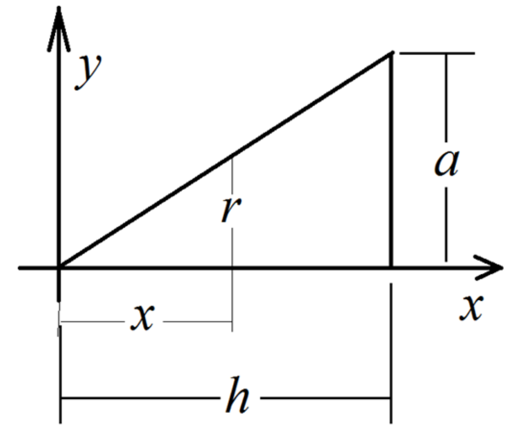


(b)

Problems (Center of Gravity)

(a)





From figure, r=(a/h) x*

$$V = \int dV = \int_0^h \pi r^2 dx = \int_0^h \pi (\frac{a}{h})^2 x^2 \, dx$$

$$\bar{x}V = \int \acute{x} dV = \int_0^h x \, \pi (\frac{a}{h})^2 x^2 dx$$

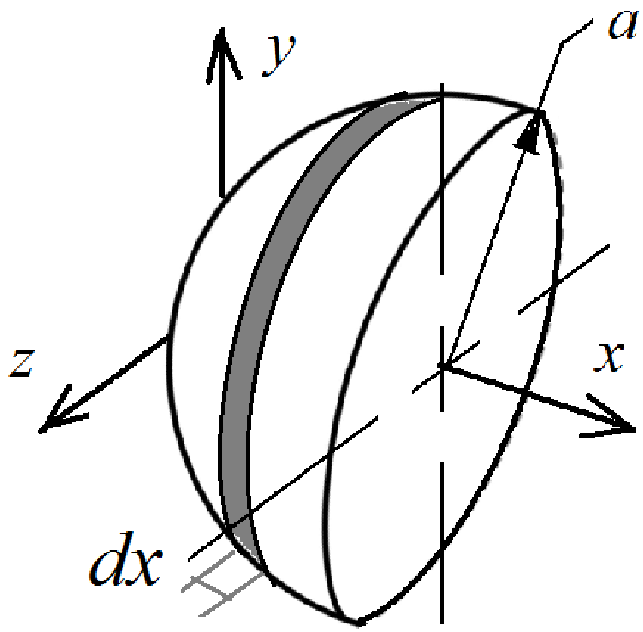
Due to symmetry :

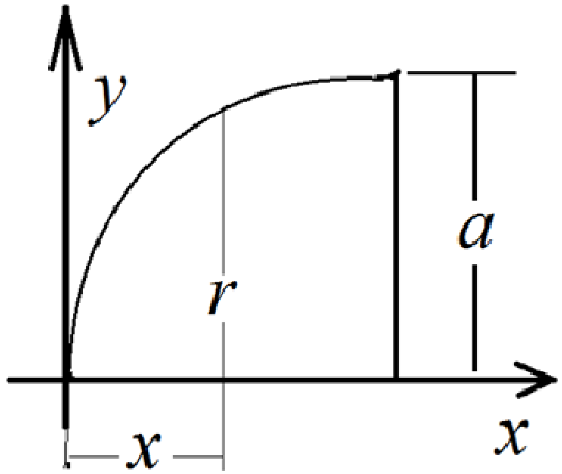
$$\bar{y}V = \int \acute{y} dV = \int_0^h 0 \times \pi (\frac{a}{h})^2 x^2 dx = 0$$

$$\bar{z}V = \int \acute{z} dV = \int_0^h 0 \times \pi (\frac{a}{h})^2 x^2 dx = 0$$

Problems (Center of Gravity)

(b)





From figure,

$$r^2 = a^2 - (x-a)^2$$

$$V = \int dV = \int_0^a \pi r^2 dx = \int_0^a \pi[a^2 - (x-a)^2 dx$$

$$\bar{x}V = \int \acute{x} dV = \int_0^a x\,\pi[a^2 - (x-a)^2 dx$$

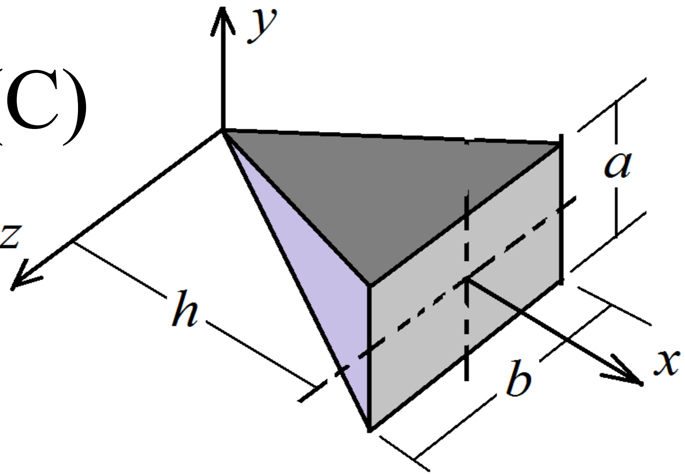
Due to symmetry :

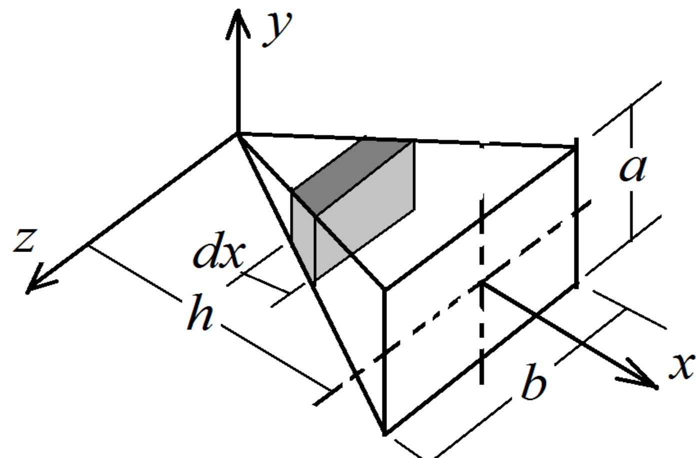
$$\bar{y}V = \int \acute{y} dV = 0$$

$$\bar{z}V = \int \acute{z} dV = 0$$

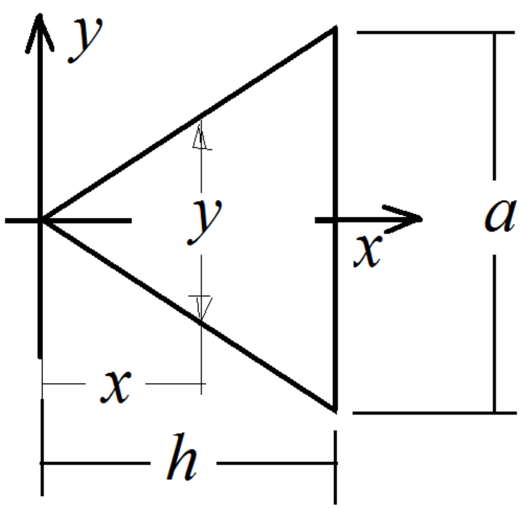
Problems (Center of Gravity)

(C)

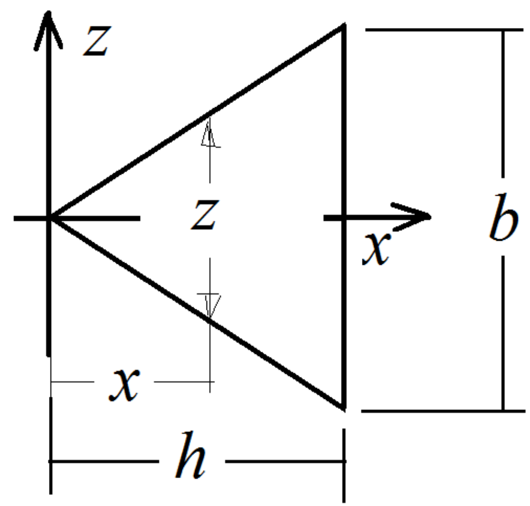




$dV = zy\, dx;$ $[z=y=f(x)]$



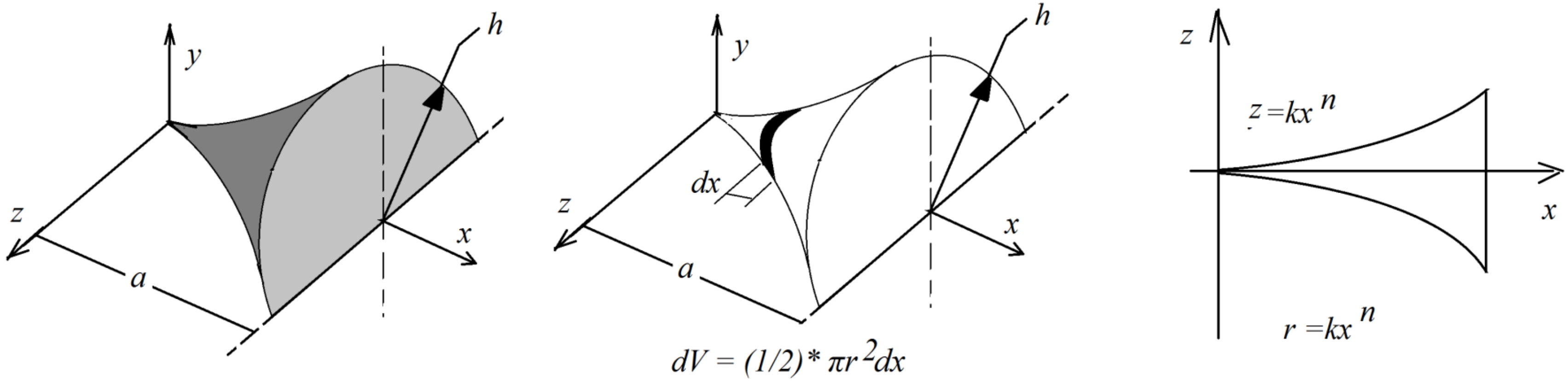
$y = (a/h)*x$



$z = (b/h)*x$

Problems (Center of Gravity)

Example -11: Determine the centroid for the solid shown in the figure.



$$V = \int dV = \int_0^a (\frac{1}{2})\pi r^2 dx = \int_0^a \pi\, k^2 x^{2n}\, dx$$

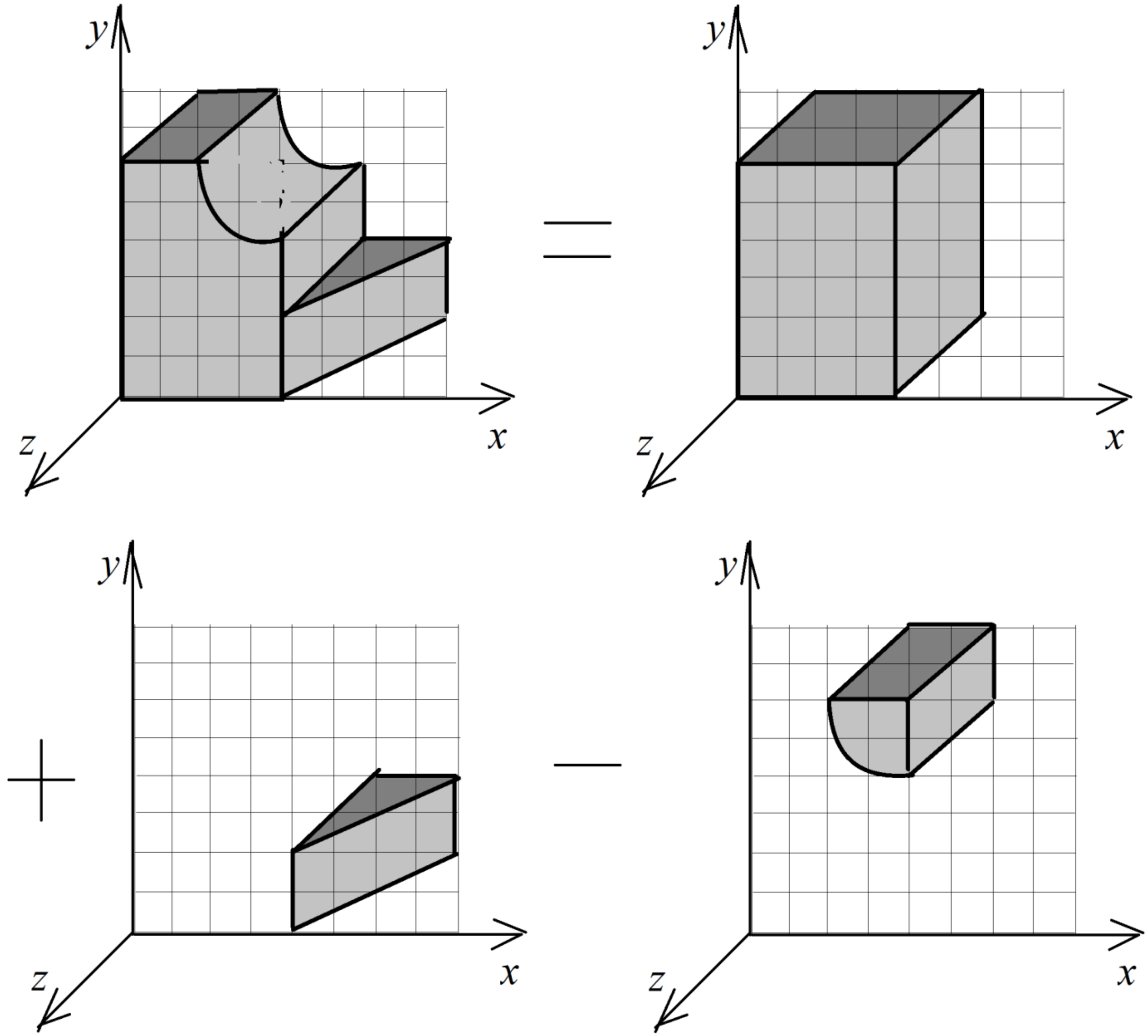
$$\bar{x}V = \int \tilde{x} dV = \int_0^a x\,(\frac{1}{2})\pi\, k^2 x^{2n} dx$$

$$\bar{y}V = \int \tilde{y} dV = \int_0^a \frac{4r}{3\pi} \times (\frac{1}{2})\pi\, k^2 x^{2n} dx = 0$$

$$\bar{z}V = \int \tilde{z} dV = 0$$

Problems (Center of Gravity)

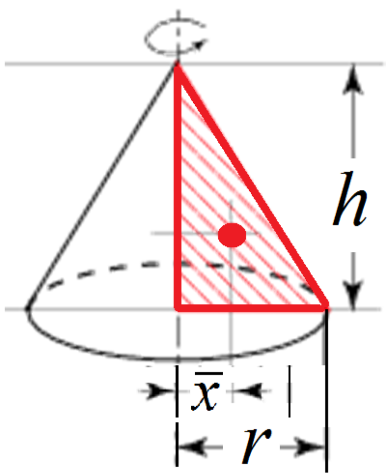
Example -12: Determine the centroid for the solid shown in the figure. (small Segment = 10 unit)



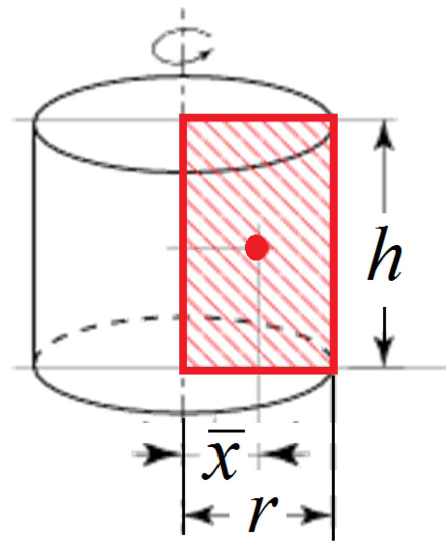
Pappus-Guldinus Theorem

- The volume of a solid of revolution is equal to the area of the generating surface times the distance traveled by the centroid of the area while the volume is being generated.

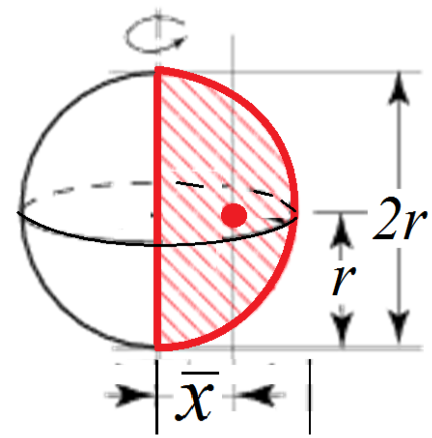
$$V = A * \delta$$



$x = r/3$
$\delta = 2\pi x$
$A = (1/2)rh$



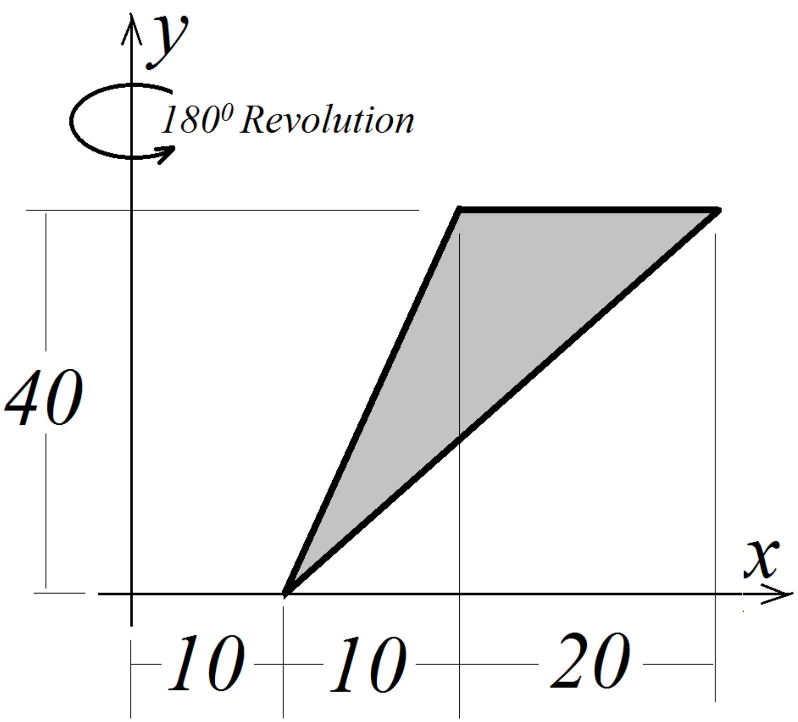
$x = r/2$
$\delta = 2\pi x$
$A = rh$



$x = (4r)/(3\pi)$
$\delta = 2\pi x$
$A = (1/2)\pi r^2$

Problems (Center of Gravity)

Example -13: Determine the volume of revolution if the area shown is revolved 180^0 with respect to y-axis.



$$V = A*\delta$$

$$A = (½)[30*40 - 10*40]$$

$$= 400 \quad (sq.\ unit)$$

$$\bar{x} = \frac{\left[\left(\frac{1}{2}\right)\times 30\times 40\right]\times 20 - \left[\left(\frac{1}{2}\right)\times 30\times 40\right]\times \frac{40}{3}}{\left[\left(\frac{1}{2}\right)\times 30\times 40\right] - \left[\left(\frac{1}{2}\right)\times 30\times 40\right]}$$

$$= 23.33 \quad (unit)$$

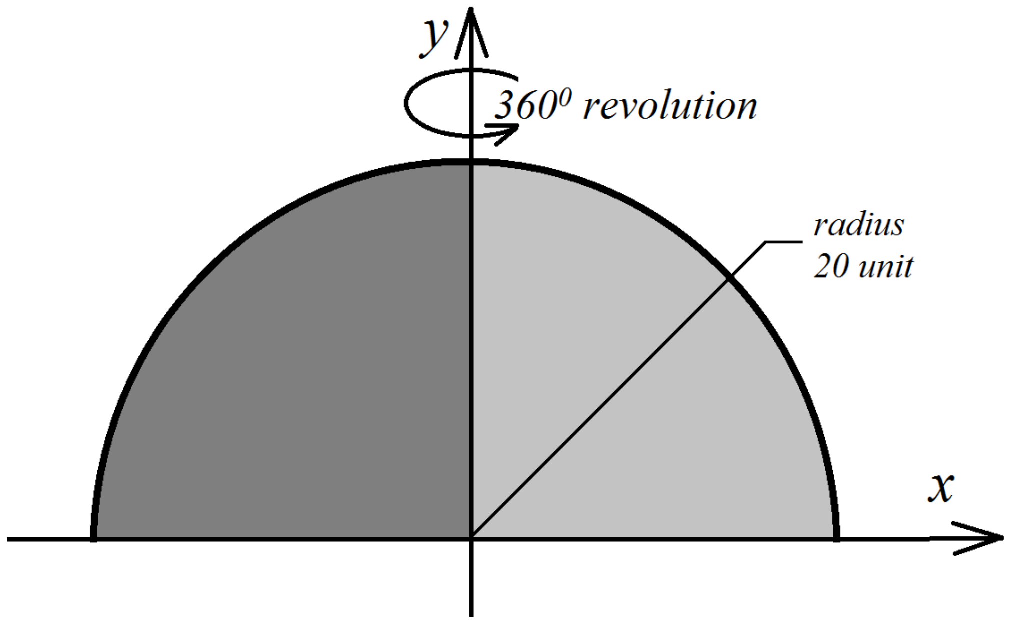
$$\delta = \pi \times 23.33 = 73.3 \quad (unit)$$

$$So,\ V = A*\delta = 400*73.3$$

$$= 29321.6 \quad (cubic\ unit)$$

Problems (Center of Gravity)

Example -14: Determine the volume of revolution if the area shown is revolved 360^0 with respect to y-axis.



$V = A*\delta$

$A = (1/4)\ \pi*20^2$ (sq. unit)

$\delta = 2\pi \times (4*20)/(3*\pi)$ (unit)

So, $V = A*\delta$

$= 16755.2$ (cubic unit)

?