Mechanics of Solids

What is solid mechanics: (Difference with Engineering mechanics)

Solid mechanics	Engineering mechanics
deals with body under force considering physical deformation	deals with body under force without deformation consideration

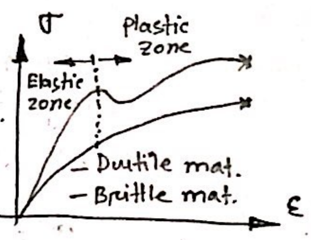
Stress: → force per unit area (Difference from pressure?) — pressure: scaler quantity.
→ Vector quantity.
→ Depends on applied force and body configuration

Type of stress: ⓐ Normal stress (Perpendicular to area)
ⓑ ~~Parallel~~ shear stress (parallel to area)

Strength: → Maximum stress that a body can withstand.
→ Depends on material (Basically)
→ Do not depends on applied force (Load)
→ failure occures when stress > strength.

Strain: → Deformation per unit length. $\varepsilon = \frac{\delta}{L}$

Stress-strain diagram:



(a) Elastic deformation: allows body to come back.
(b) Plastic deformation: permanent deformation occure.

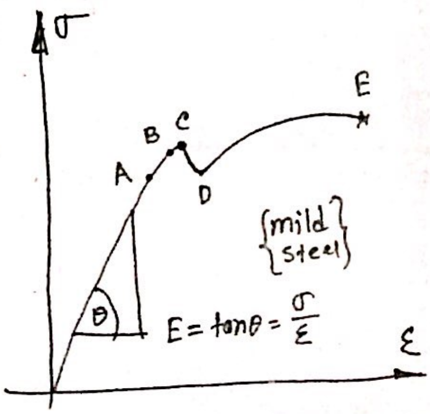
Some terminology: (From σ-ε diagram)

ⓐ proportional limit: (A) upto where σ is proportional to ε.

ⓑ Elastic ~~mit~~ limit: (B) Beyond which point plastic deformation starts.

ⓒ Yield point: (C) σ-ε plot is horizontal,
→ No stress is required for strain.

ⓓ Ultimate strength: (E) Maximum stress a body can withstand.



(e) Modulus of Elasticity: → slope of σ-ε diagram upto proportional lim

(f) Modulus of Resilience: → Energy absorbed per unit volume
→ Area under σ-ε curve upto proportional limit
→ $U = \frac{1}{2}\sigma \cdot \varepsilon$ (Mathmatical Expression)

(g) Modulus of toughness: → Energy absorbed before complete failure.

(h) Hardness, toughness, strength, resilience are not same.

<u>Yield point in σ-ε diagram of brittle material:</u>

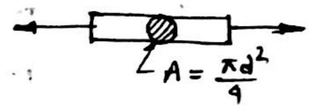
→ No fixed and well defined yield point.

→ Offset method are used (0.2% or 0.5%) 2% [means $\varepsilon = 0.002$]

<u>Problem: (1)</u> A rod of 20 mm diameter is to be elongated by an axial force of 50 N. Determine the stress type and magnitude.

<u>Solution:</u> Normal (Tensile) stress.

$$\sigma = \frac{F}{A} = \frac{50}{\pi\left(\frac{20^2}{4}\right)} = 0.16 \text{ MPa}$$

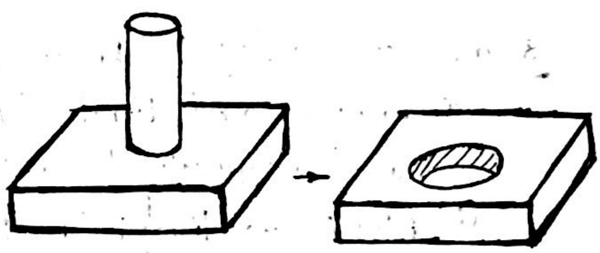


$\boxed{\sigma \text{ in MPa}, \ F \text{ in N} \quad l, d \text{ in mm}}$

<u>Problem: (2)</u> A 10 mm thick metal plate is to be punced to create a 20 mm dia hole (through). Determine the stress type and magnitude of stress. (F = 10 kN)

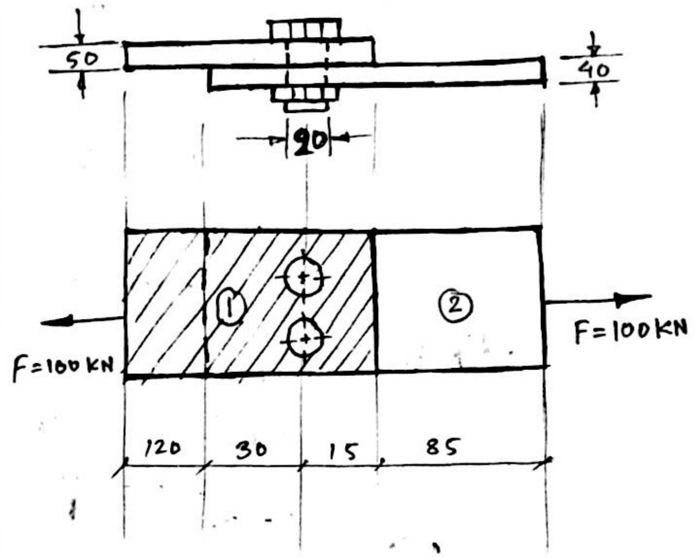
<u>Solution:</u> shear (Direct) stress.

$$\tau = \frac{F}{A} = \frac{10 \times 10^3}{(\pi \times 20) \times 10}$$
$$= 15.9 \text{ MPa}.$$



Problem (3) Determine all the stresses in the plates and rivets/bolts.

Solution:



(a) Tension in plates:

plate ①: $\sigma = \dfrac{100\times10^3}{(170-2\times20)\times50} = 15.4$ MPa

plate ②: $\sigma = \dfrac{100\times10^3}{(170-2\times20)\times40} = 19.2$ MPa

(b) Shear in rivets:

$\tau = \dfrac{100\times10^3}{2\times(\pi\times\frac{20^2}{4})} = 159.2$ MPa

(c) Compression in plate and rivets: (Bearing stress)

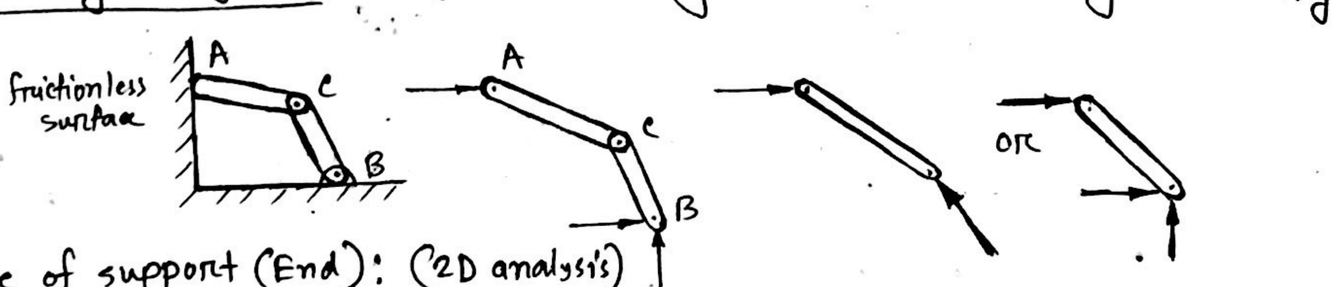
plate ①: $\sigma = \dfrac{100\times10^3}{2\times(50\times20)} = 50$ MPa

plate ②: $\sigma = \dfrac{100\times10^3}{2\times(40\times20)} = 62.5$ MPa

(d) shear in plates (Tearing stress): Plate ①: $\tau = \dfrac{100\times10^3}{4\times(15\times50)} = 33.3$ MPa

plate ②: $\tau = \dfrac{100\times10^3}{4\times(30\times40)} = 20.8$ MPa.

Free body diagram: Diagram showing all the forces acting on a body.



Type of support (End): (2D analysis)

(a) Fixed: Two (2) forces + One (1) moment.

(b) Hinged: Two (2) Forces

(c) Roller: One (1) Forces.

Statically two types of structures:

(a) Determinate

(b) Indeterminate.

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Problem: (4) Compute the shearing stress in the pin.

Solution:

$\circlearrowleft + \sum M_B = 0;\ F_c \times 250 - 34.64 \times 200 = 0$

$\Rightarrow F_c = 27.7\ KN\ (\uparrow)$

$\xrightarrow{+} \sum F_x = 0;\ F_B^x + 34.64 = 0$

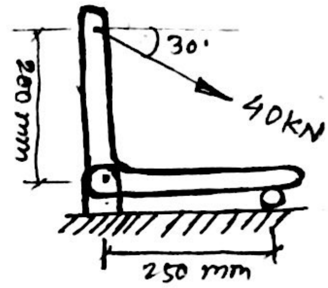
$\Rightarrow F_B^x = -34.64\ (\rightarrow)$

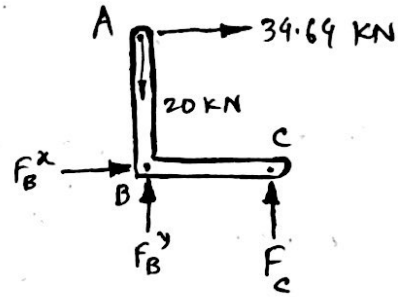
$= 34.64\ (\leftarrow)$

$+\uparrow \sum F_y = 0;\ F_B^y + 27.7 - 20 = 0$

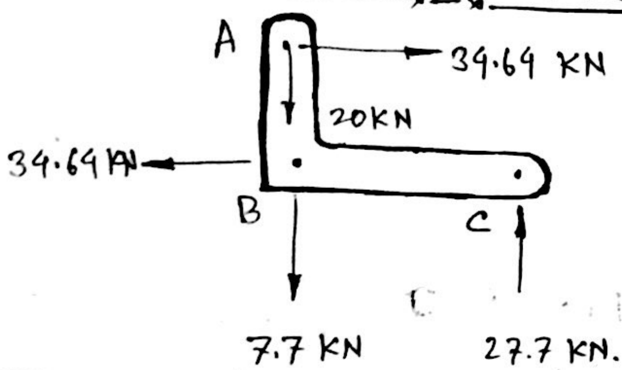
$\Rightarrow F_B^y = -7.7\ (\uparrow)$

$= 7.7\ (\downarrow)$





Assumed free body diagram.

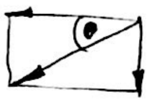


Actual free body diagram

Total force at point B (pin): $F_B = \sqrt{(F_B^x)^2 + (F_B^y)^2} = 35.5\ KN$

direction of F_B: $\theta = \tan^{-1}\left(\frac{F_B^y}{F_B^x}\right) = \tan^{-1}\left(\frac{7.7}{34.64}\right) = 12.53°$

So, $\boxed{\vec{F_B} = 35.5\ KN\ \ 12.53°}$



Shear stress: $\tau = \frac{F}{A} = \frac{35.5 \times 4}{2 \times \pi \times 20^2} = 56.5\ MPa$ (Ans).

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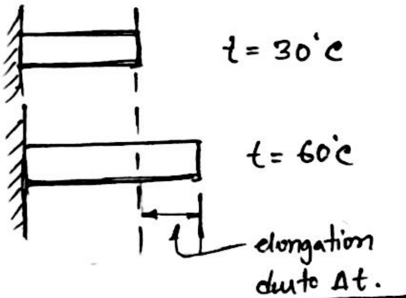
Hook's law: $\sigma \propto \varepsilon$; $\sigma = E\varepsilon$; $E = \frac{\sigma}{\varepsilon} = \frac{FL}{A\delta}$

$\therefore \boxed{\delta = \frac{FL}{AE}}$

Thermal stress:

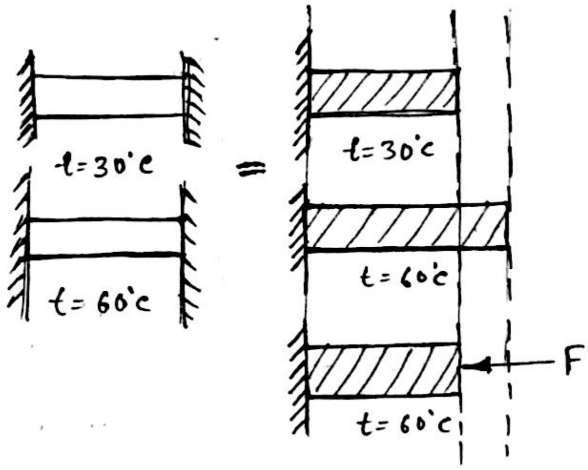
→ stress developed due to change in temperature and restriction to elongation.

→ If the temperature changes and the body is free to deform then no thermal stress is developed.

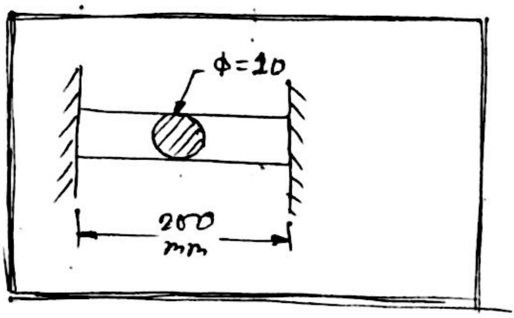


$\boxed{\delta_t = \alpha L \Delta t}$

α = coefficient of thermal expansion. (m/m°c)



Problem:(5) A metal bar is held by two supports at 30°c. Then the temperature changes to 100°c find out the force that the supports exerts on the bar. The thermal expansion coefficient and modulus of elasticity are ~~...~~ 10^{-7} m/m°c and 200 GPa respectively.



Solution: Net deformation is zero.

$\therefore \delta_t + \delta_F = 0$

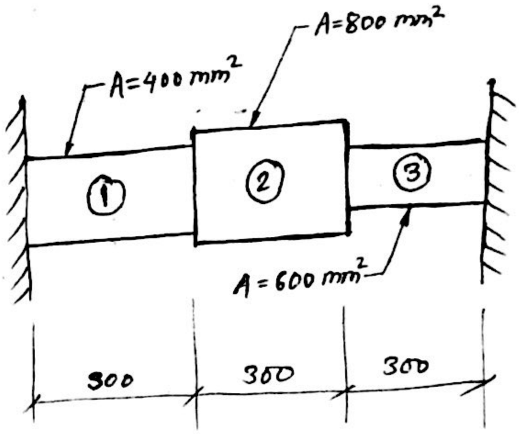
$\Rightarrow \alpha L(\Delta t) + \frac{FL}{AE} = 0$

$\Rightarrow 10^{-7} \times 200 \times (100-30) + \frac{F \times 200}{200 \times 10^3 \times \pi \times 5^2} = 0$

$\Rightarrow F = -110$ N

$= 110$ N (compressive)

Problem: (6) Determine the thermal stress in every member. Temperature changes to $-10^\circ C$ from $40^\circ C$.



$E = 200$ GPa

$\alpha = 10^{-6} \frac{m}{m^\circ C}$

For all member

Solution: $\delta_t = (\alpha L \Delta t)_1 + (\alpha L \Delta t)_2 + (\alpha L \Delta t)_3$

$= \alpha \Delta t (L_1 + L_2 + L_3)$

$= 10^{-6} \times (-50) \times 900 = -0.045$ mm

$$\delta_p = \left(\frac{FL}{AE}\right)_1 + \left(\frac{FL}{AE}\right)_2 + \left(\frac{FL}{AE}\right)_3$$

$$= \frac{FL}{E}\left(\frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3}\right)$$

$$= \frac{F \times 300}{200 \times 10^3}\left(\frac{1}{400} + \frac{1}{800} + \frac{1}{600}\right) = 8.125 \times 10^{-6} F.$$

Now, $\delta_t + \delta_p = 0$.

$\Rightarrow -0.045 + 8.125 \times 10^{-6} F = 0$

$\Rightarrow F = 5.54$ KN (Tensile)

stress:

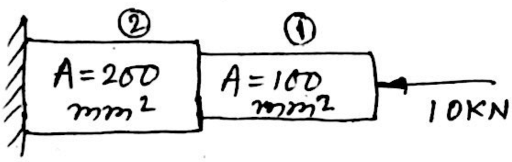
Section①: $\sigma_t = \frac{5.54 \times 10^3}{400} = 13.8$ MPa

Section(ii): $\sigma_t = \frac{5.54 \times 10^3}{800} = 6.9$ MPa

Section(iii): $\sigma_t = \frac{5.54 \times 10^3}{600} = 10.4$ MPa

Problem: (7) Find the stresses in each section:

(a)

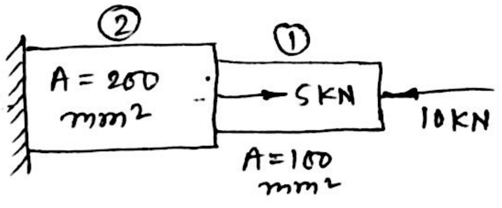


Solution: $P_1 = P_2 = 10$ KN

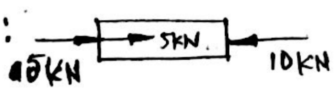
section①: $\sigma = \frac{10 \times 10^3}{100} = 100$ MPa

Section (ii): $\sigma = \frac{10 \times 10^3}{200} = 50$ MPa.

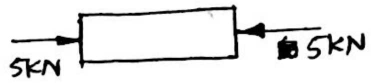
(b)



section ①:



section (ii)



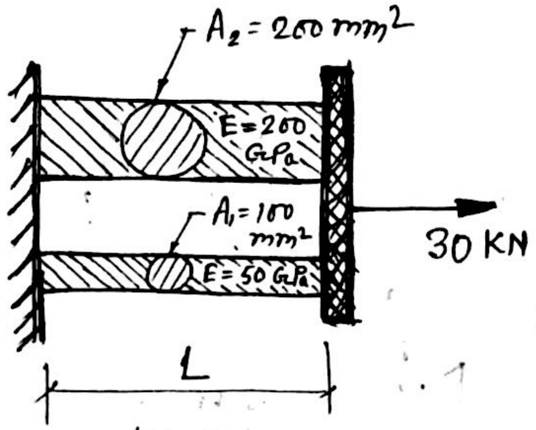
Solution: $P_1 \neq P_2$

section①: $\sigma = \frac{10 \times 10^3}{100} = 100$ MPa

Section(ii): $\sigma = \frac{5 \times 10^3}{200}$

$= 25$ MPa

(c)



Solution: $P_1 \neq P_2$; $\boxed{P_1 + P_2 = 30}$ KN

$$\delta_1 = \delta_2 \Rightarrow \left(\frac{PL}{AE}\right)_1 = \left(\frac{PL}{AE}\right)_2$$

$$\Rightarrow P_1 = \left(\frac{A_1}{A_2}\right)\left(\frac{E_1}{E_2}\right) \cdot P_2$$

$$\Rightarrow P_1 = (0.5)(0.25) \cdot P_2$$

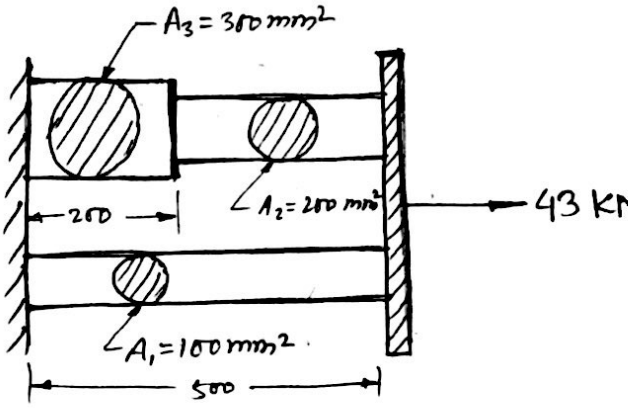
$$\Rightarrow P_2 = 8P_1 \; ; \; \boxed{P_1 + P_2 = 30\ KN}$$

$\therefore P_1 = 3.33$ KN ; $P_2 = 26.67$ KN.

$$\sigma_1 = \left\{\frac{3.33 \times 10^3}{100}\right\} = 33.3\ MPa$$

$$\sigma_2 = \left\{\frac{26.67 \times 10^3}{200}\right\} = 133.35\ MPa$$

(d)



Solution: $\delta_1 = \delta_2 + \delta_3$; $\boxed{P_2 = P_3}$

$$\Rightarrow \left(\frac{PL}{AE}\right)_1 = \left(\frac{PL}{AE}\right)_2 + \left(\frac{PL}{AE}\right)_3$$

$$\Rightarrow \left(\frac{P_1 \times 500}{100}\right) = \left(\frac{P_2 \times 300}{200}\right) + \left(\frac{P_2 \times 200}{300}\right)$$

$$\Rightarrow P_2 = \left(\frac{30}{13}\right) P_1 \qquad \boxed{P_1 + P_2 = 43}$$

$\therefore P_1 = 13$ KN; $P_2 = 30$ KN.

$$\sigma_1 = \frac{13 \times 10^3}{100} = 130\ MPa$$

$$\sigma_2 = \frac{30 \times 10^3}{200} = 150\ MPa$$

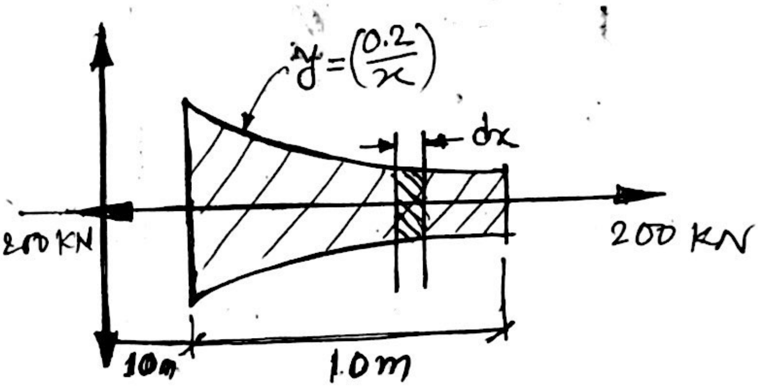
$$\sigma_3 = \frac{30 \times 10^3}{300} = 100\ MPa$$

* $\boxed{\text{Home work}}$ determine the total elongation if a bar like bellow is subjected to 200 KN Force. $[E = 200\ GPa]$

elementary elongation: $d\delta = \left(\frac{FL}{AE}\right)$

$$\Rightarrow d\delta = \frac{200\times10^{3}\,dx}{2\left(\frac{0.2}{x}\right)\times0.01\times200\times10^{9}}$$

$$= \left(\frac{25x}{100000}\right)dx$$



Thickness of bar = 10 mm.

$\therefore$ Total elongation: $\delta = \int_{10}^{20} d\delta$

$$= \frac{25}{100000}\int_{10}^{20} x\,dx$$

$$= \frac{25}{2\times100000}\left[20^2 - 10^2\right]$$

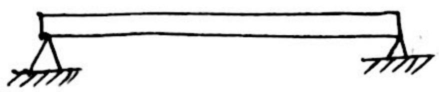
$$= 0.0375\ m = 37.5\ mm.$$

(Ans)

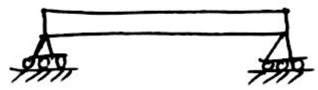
* Work out problem: 201 from book (Singer).

Beam and type of beam:

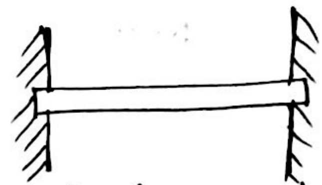
- A structural member that carries transverse load.
- Beam are classified according to their end/support condition.



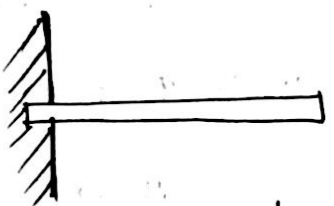
(a) Hinged support



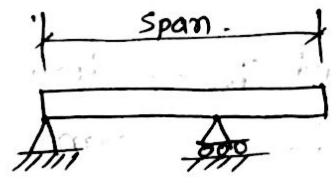
(b) Simply support



(c) fixed support.

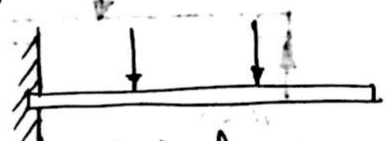


(d) Cantelever beam

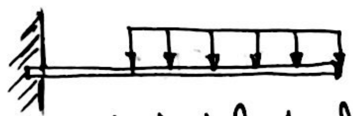


(e) Overhanging beam.

- A beam can have two different kind support at two different end.
- Type of load:



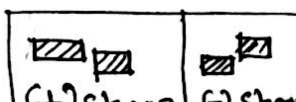
(a) Point load



(b) Distributed load.

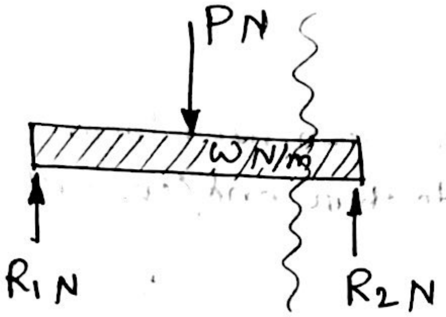
Shear force in beam:

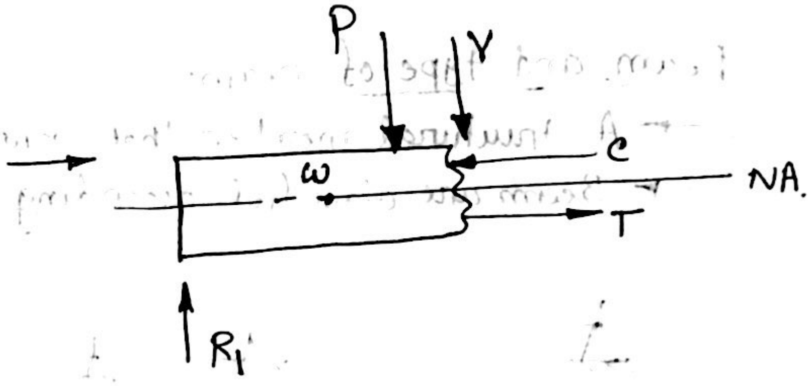
- When a beam is subjected to a transverse loading shear force act at every section of the beam.
- The sign convension for shear force are:



Determination of shear force:

- consider a beam of weight w N/m is subjected to loading.
- From free body diagram of any segment of beam we can write:-

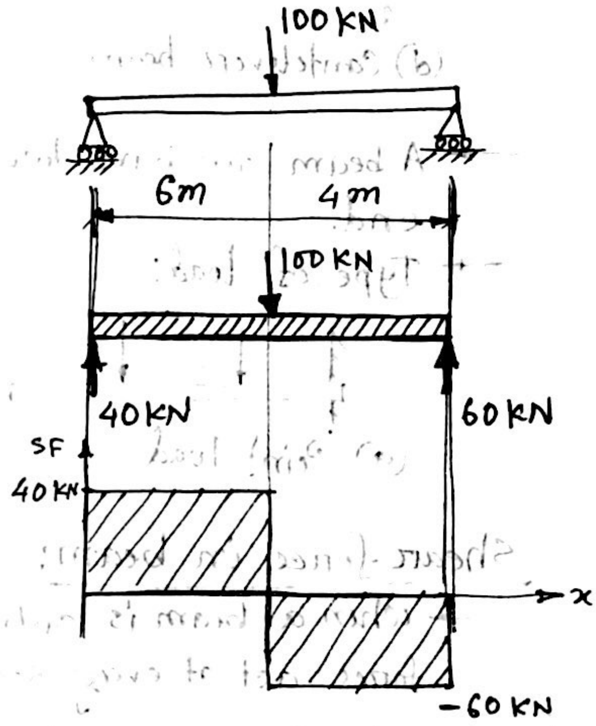




$$\boxed{V = R_1 - P - W}$$

Here no leter represents Force but they represents Force due to loading type

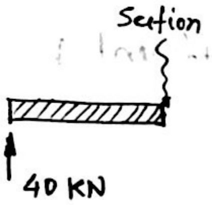
Problem: (8) Find the equation and draw the shear force for the following beam.



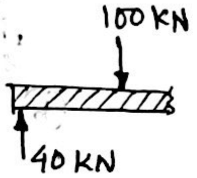
Solution:

For, $0 < x < 6$:

$V = 40$ KN



For, $6 < x < 10$:



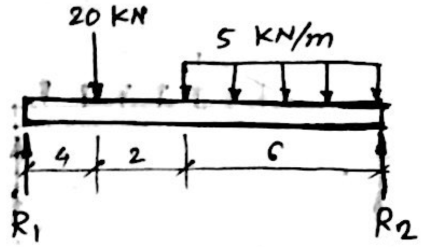
$V = 40 - 100$

$= -60$ KN.

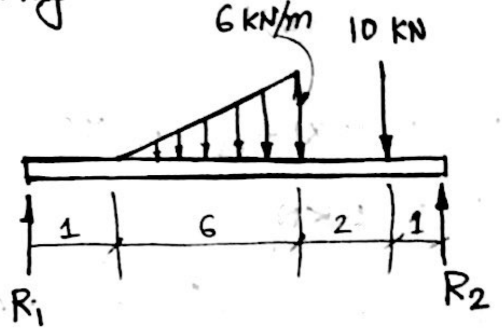
$$\therefore V = \begin{cases} 40 \; ; & 0 < x < 6 \\ -60 \; ; & 6 < x < 10 \end{cases}$$

Problem: (9) Draw shear force and bending moment diagram.

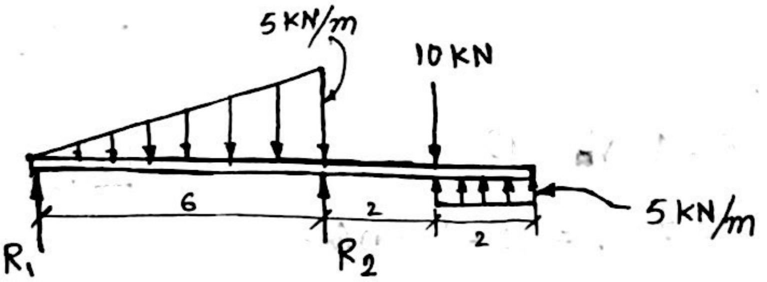
(a)



(b)



Assignment (1) (c)



Solution: (a)

Determination of Reactions:

$$R_1 \times 12 = 20 \times 8 + (5 \times 6) \times 3$$

$$\Rightarrow R_1 = 20.83 \text{ KN.}$$

$$R_2 \times 12 = 20 \times 4 + (5 \times 6) \times 9$$

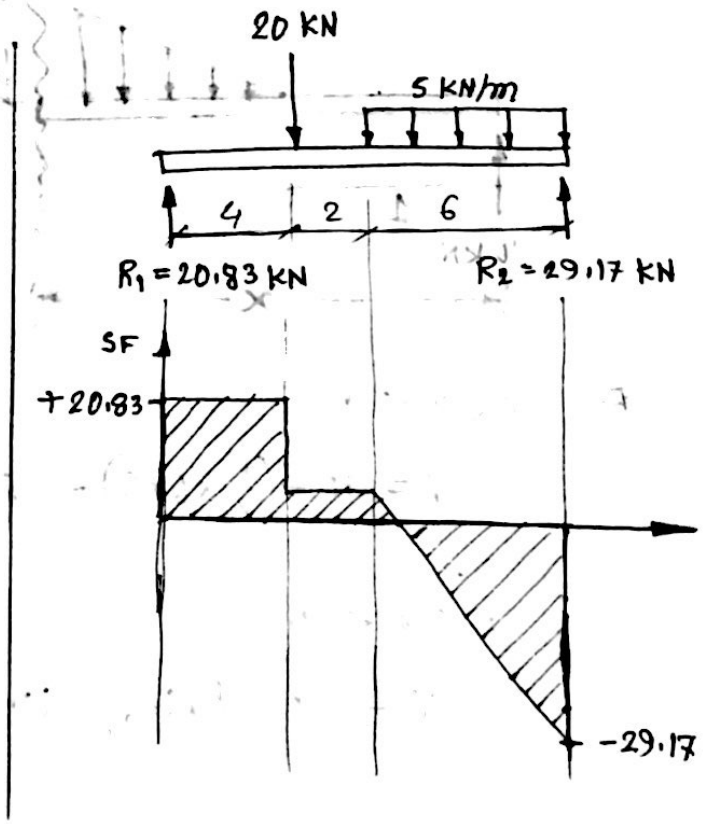
$$\Rightarrow R_2 = 29.17 \text{ KN.}$$

For $0 < x < 4$: $V = 20.83$

For $4 < x < 6$: $V = 20.83 - 20$

For $6 < x < 12$: $V = 20.83 - 20 - 5(x-6)$

$$\Rightarrow V = 30.83 - 5x$$



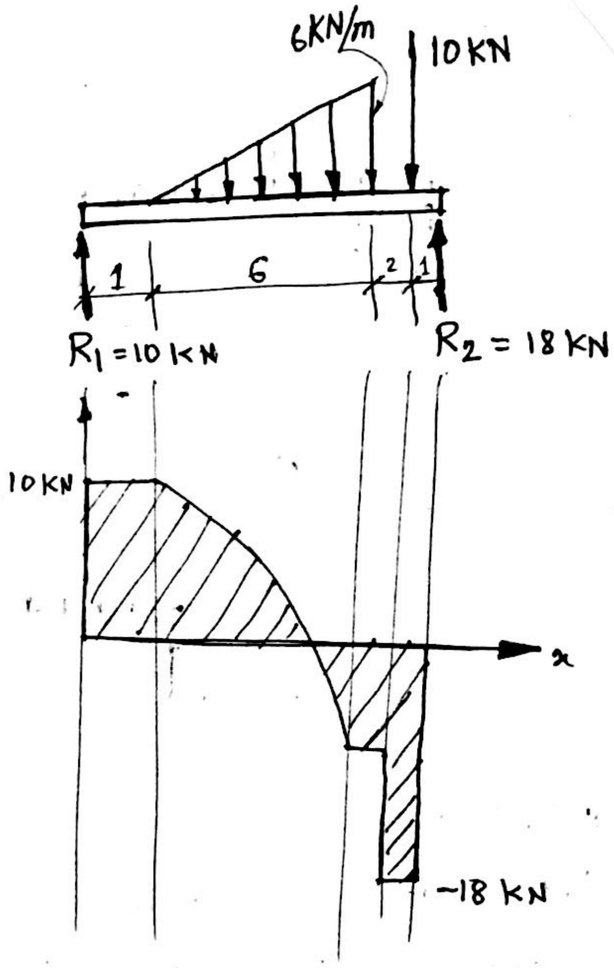
Solution: (b)

$R_1 \times 10 = 10 \times 1 + \left(\frac{1}{2} \times 6 \times 6\right) \times 5$

$\Rightarrow R_1 = 10\ KN.$

$R_2 \times 10 = 10 \times 9 + \left(\frac{1}{2} \times 6 \times 6\right) \times 5$

$\Rightarrow R_2 = 18\ KN.$

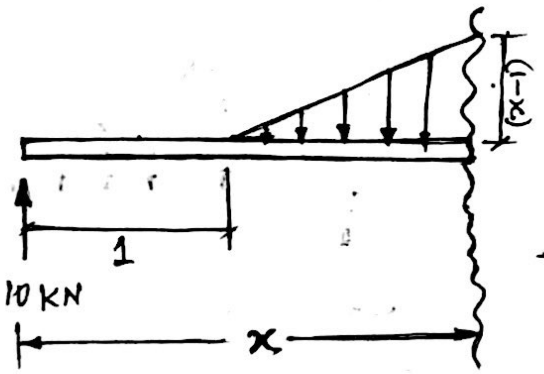


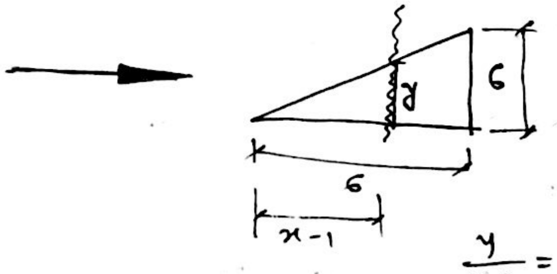
Equation:

For, $0 < x < 1$, $V = 10$,

For, $1 < x < 7$, $V = 10 - \frac{1}{2}(x-1)(x-1)$

$= 10 - \frac{(x-1)^2}{2}$





$\frac{y}{x-1} = \frac{6}{6}$

$\Rightarrow y = (x-1)$

For, $7 < x < 9$,

$V = 10 - \left(\frac{1}{2} \times 6 \times 6\right) = -8$

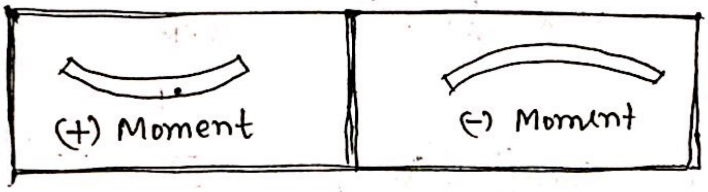
For, $9 < x < 10$,

$V = 10 - \left(\frac{1}{2} \times 6 \times 6\right) - 10 = -18$

Bending moment in beam:

→ When a beam is subjected to load every section feels different moment.

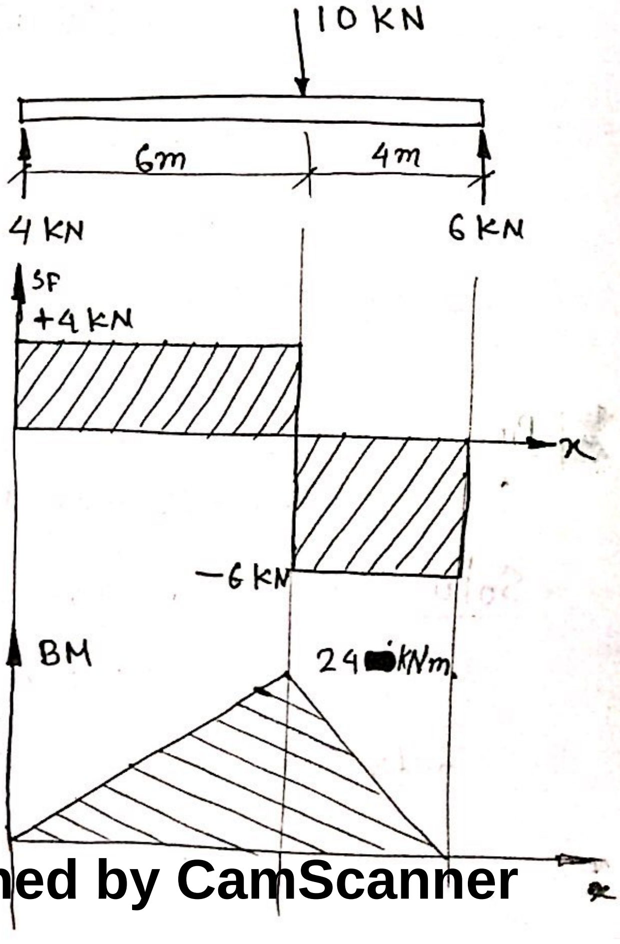
→ The sign convension for bending moment is



→ Bending moments are found in the same way of finding shear force exept it has to be the moment about the cross-section.

Problem:(10) Draw the bending moment diagram of the beam.

Solution:



For $0 < x < 6$,

$$M = 4x$$

For $6 < x < 10$,

$$M = 4x - 10(x-6)$$
$$= 60 - 6x$$

Conclusion:

① There is always a point when shear force is zero and bending moment is (maximum).

② The point with maximum bending moment is known as dangerous

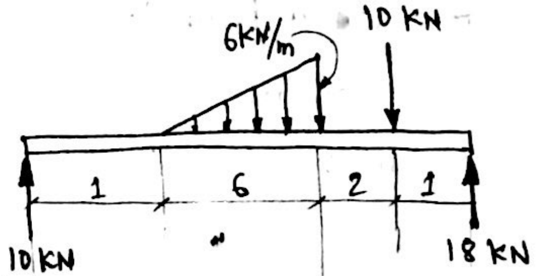
Problem:(11) Draw the bending moment diagram for the beam

Solution:

For $0<x<1$; $M = 10x$

For $1<x<7$; $M = 10x - \frac{1}{2}(x-1)^2\left(\frac{x-1}{3}\right)$

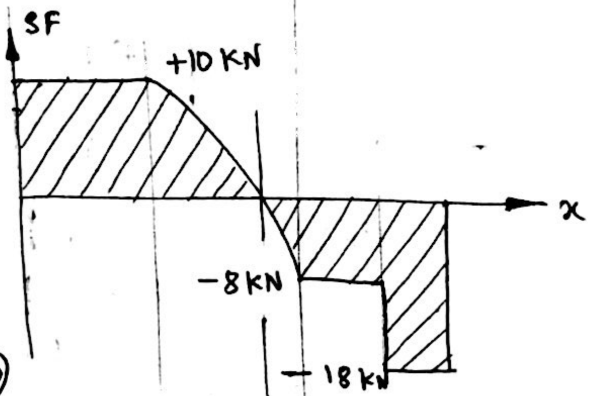
$= 10x - \frac{1}{6}(x-1)^3$



For $7<x<9$; $M = 10x - \left(\frac{1}{2}\cdot 6\cdot 6\right)(x-5)$

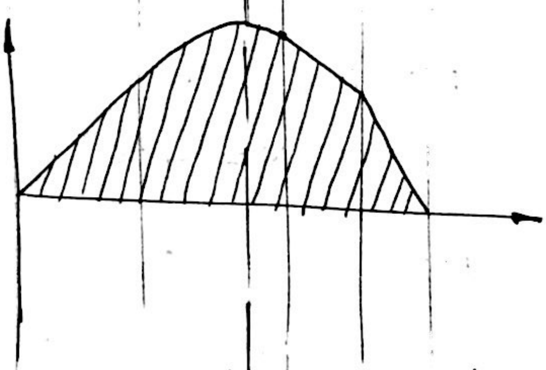
$= 10x - 18x + 90$

$= 90 - 8x$



For $9<x<10$; $M = 10x - \left(\frac{1}{2}\cdot 6\cdot 6\right)(x-5) - 10(x-9)$

$= 180 - 18x$



Problem:12 Find the value of maximum bending moment for the beam in problem:11.

Solution: Maximum bending moment is in $1<x<7$.

So, $M = 10x - \frac{1}{6}(x-1)^3$

To find x where M is maximum: $\frac{dM}{dx} = 0$

$\Rightarrow 10 - \frac{1}{2}(x-1)^2 = 0$

$\Rightarrow x = 5.47$ m.

$\therefore M_{max} = 54.7 - \frac{(4.47)^3}{6}$

$= 51.3698$

≈ 51.37 KNm

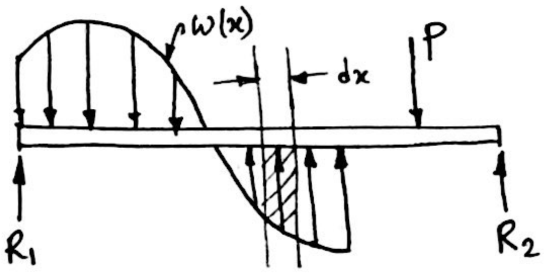
Alternate method: The maximum moment is in $1 < x < 7$.
where Moment is maximum shear force is zero.

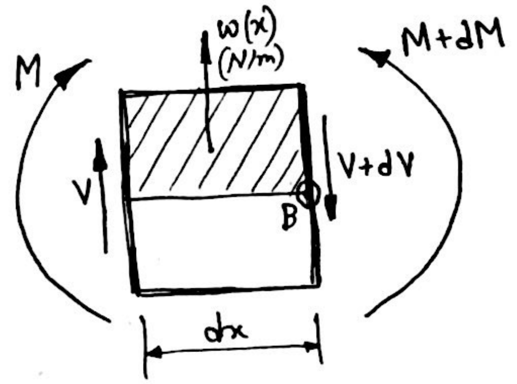
$$\therefore \quad 10 - \frac{1}{2}(x-1)^2 = 0 \; ; \quad x = 5.47.$$

So, $M_{max} = 10 \times 5.47 - \frac{1}{6}(5.47-1)^3 = 51.37 \text{ kNm}.$

<u>Relation between load, shear force and bending moment:</u>

→ Consider a beam is subjected to an arbitrary load.





→ consider an elementary length dx.

→ For static equilibrium:

$$+\uparrow \sum F_y = 0 \; ; \quad V + \omega\,dx - (V + dV) = 0 \; ; \; dV = \omega\,dx$$

$$\Rightarrow \boxed{\omega = \frac{dV}{dx} \; ; \quad V = \int \omega\,dx}$$

→ For static equilibrium:

$$+\circlearrowleft \sum M_B = 0; \quad M + (\omega\,dx)\frac{dx}{2} - (M + dM) = 0$$

$$\Rightarrow dM = V\,dx$$ neglecting product of two derivatives

$$\Rightarrow \boxed{V = \frac{dM}{dx}}$$

$$\Rightarrow \boxed{M = \int V\,dx}$$

So in summery: $\Delta V = V_2 - V_1 = \int_{x_1}^{x_2} w(x)\, dx$

$$\Rightarrow \boxed{V_2 - V_1 = (Area)_{Load}}$$

$$\Delta M = M_2 - M_1 = \int_{x_1}^{x_2} V(x)\, dx$$

$$\Rightarrow \boxed{(M_2 - M_1) = (Area)_{shear\ force}}$$

Conclusions:

→ For a point load, $w(x)$ = Undefined, this type of load is load with order negetive one (−1).

→ For an uniformly distributed load, $w(x)$ = constant, this type of load is load with order zero (0).

→ For a triangular distributed load, $w(x) = a + bx$, this type of load is load with order one (1).

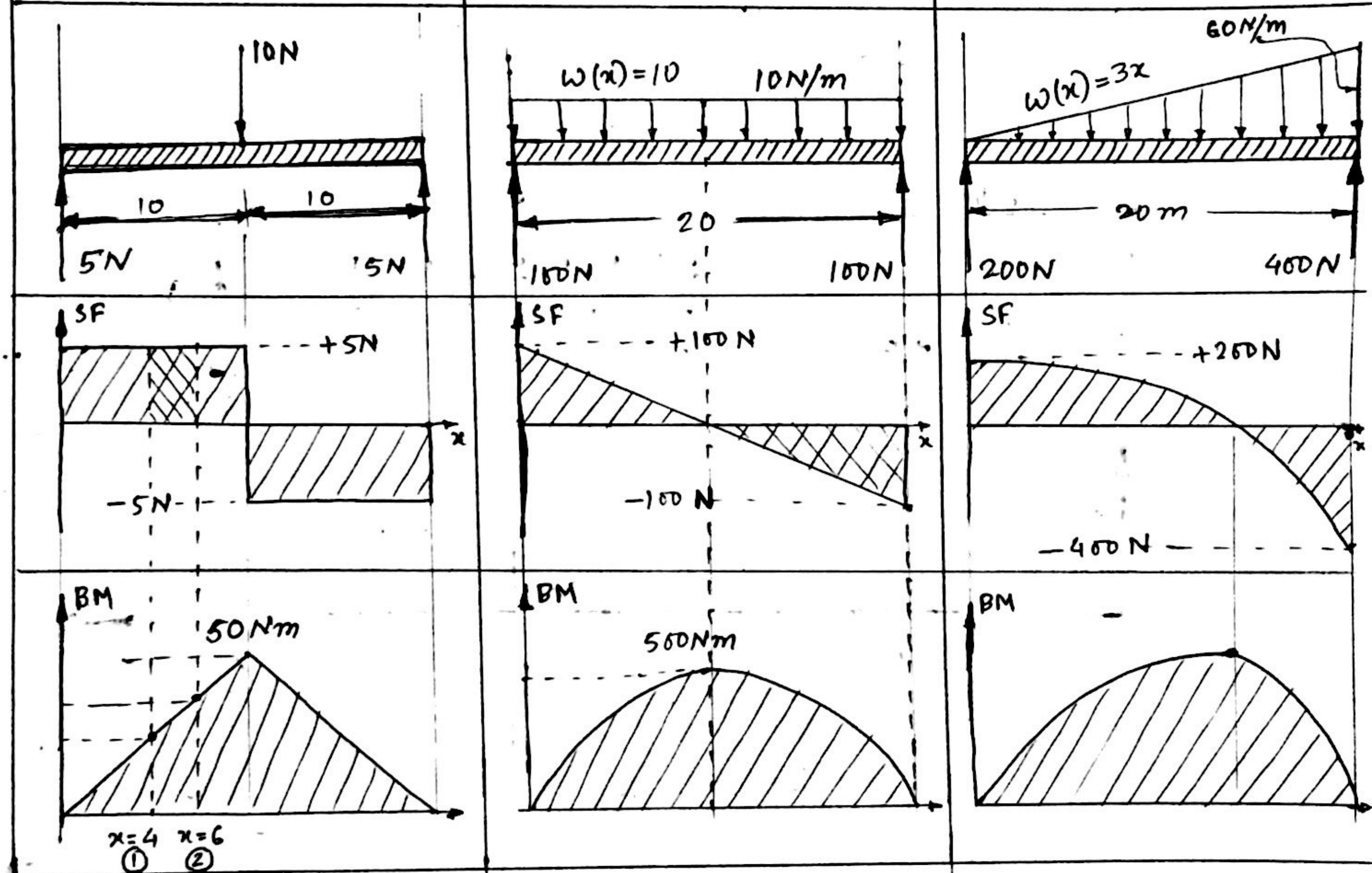
→ The order of shear force equation is always one more than order of load.

→ The order of bending moment is always two more than order of load.

$$\boxed{\begin{aligned} O_{M(x)} &= O_{V(x)} + 1 \\ O_{M(x)} &= O_{w(x)} + 2 \\ O_{V(x)} &= O_{w(x)} + 1 \end{aligned}}$$

$O_{w(x)}$, $O_{V(x)}$, $O_{M(x)}$ are order of load, shear force and bending moment respectively.

−1 order load	0 order load	+1 order load



−1 order load

$(\text{Area})_{\text{shear force}} = 5 \times 2 = 10 \text{ N·m}$

$M_2 = \frac{50}{10} \times 6 = 30 \text{ Nm}$

$M_1 = \frac{50}{10} \times 4 = 20 \text{ Nm.}$

$M_2 - M_1 = (30 - 20) \text{ Nm} = 10 \text{ Nm.}$

0 order load

$(\text{Area})_{\text{load}} = (-10) \times 10 = -100 \text{ N}$
Force downward

$V_2 = 0, \quad V_1 = 100 \text{ N.}$

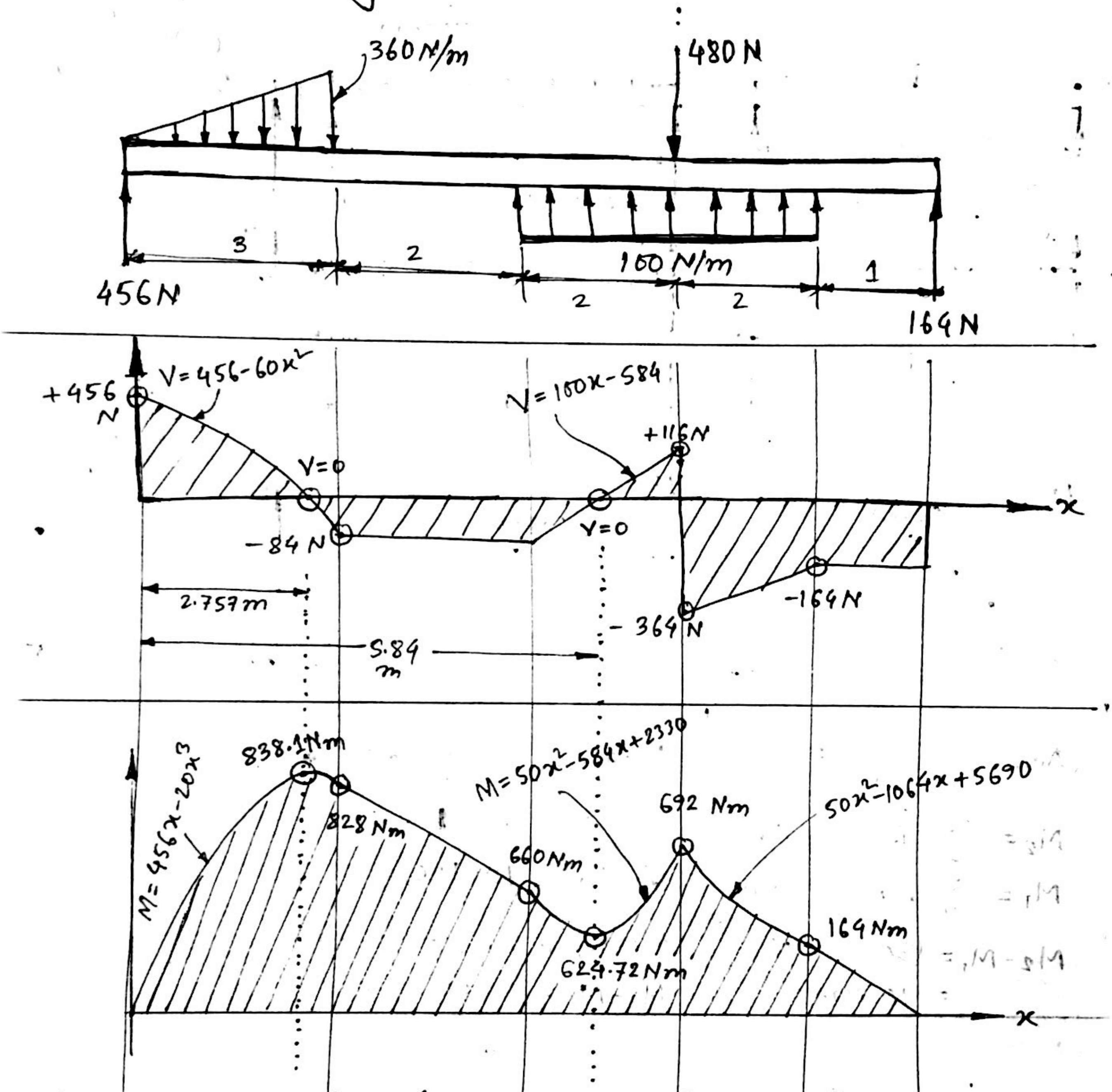
$V_2 - V_1 = (0 - 100) = -100 \text{ N}$

$(\text{Area})_{\text{shear force}} = -500 \text{ Nm}$

$M_2 - M_1 = (0 - 500) = -500 \text{ Nm}$

Shear force, bending moment diagram analysis:

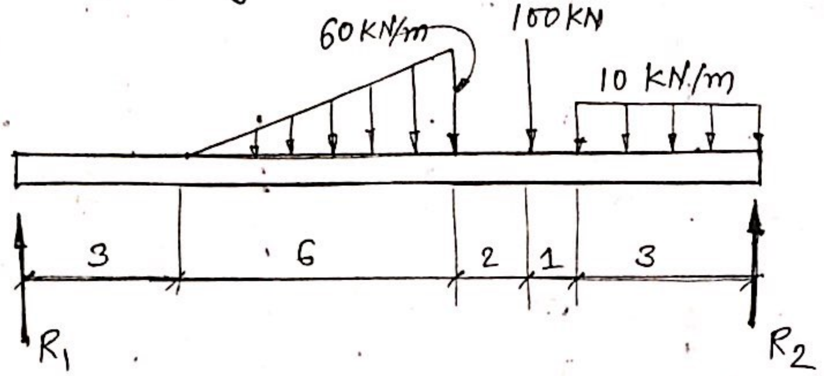
Problem:(13) Find the location and magnitude of maximum shear force and bending moment.



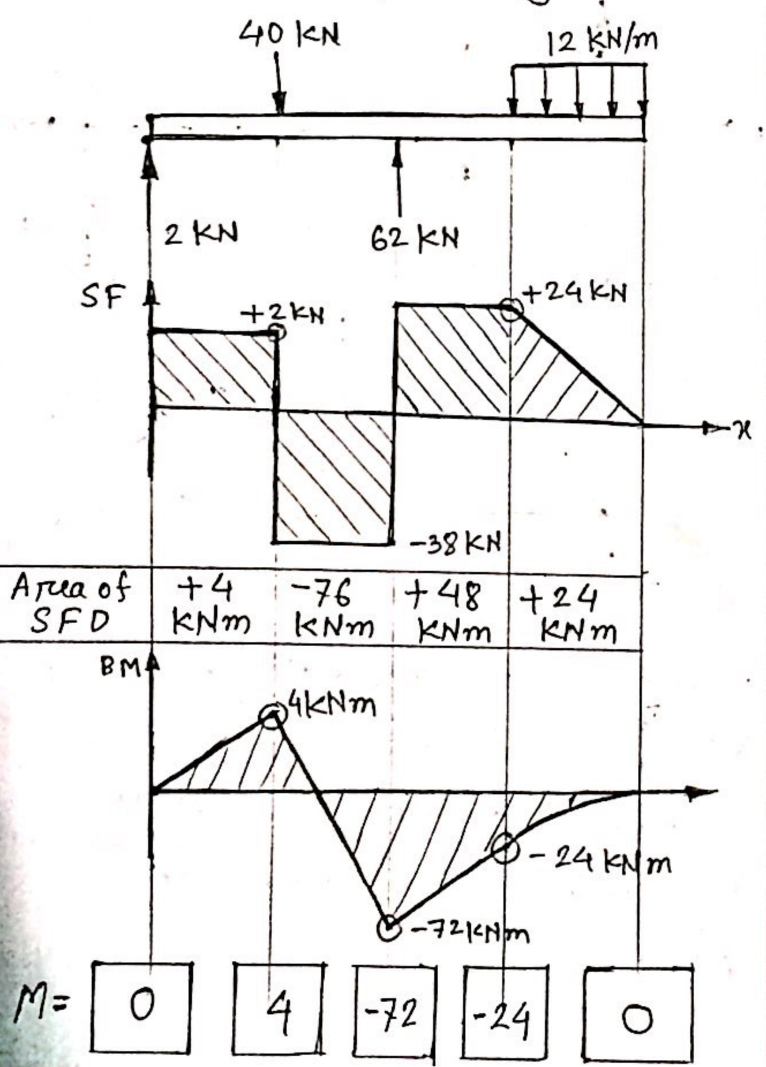
Ans: Maximum shear force = 456 N (at $x = 0$)

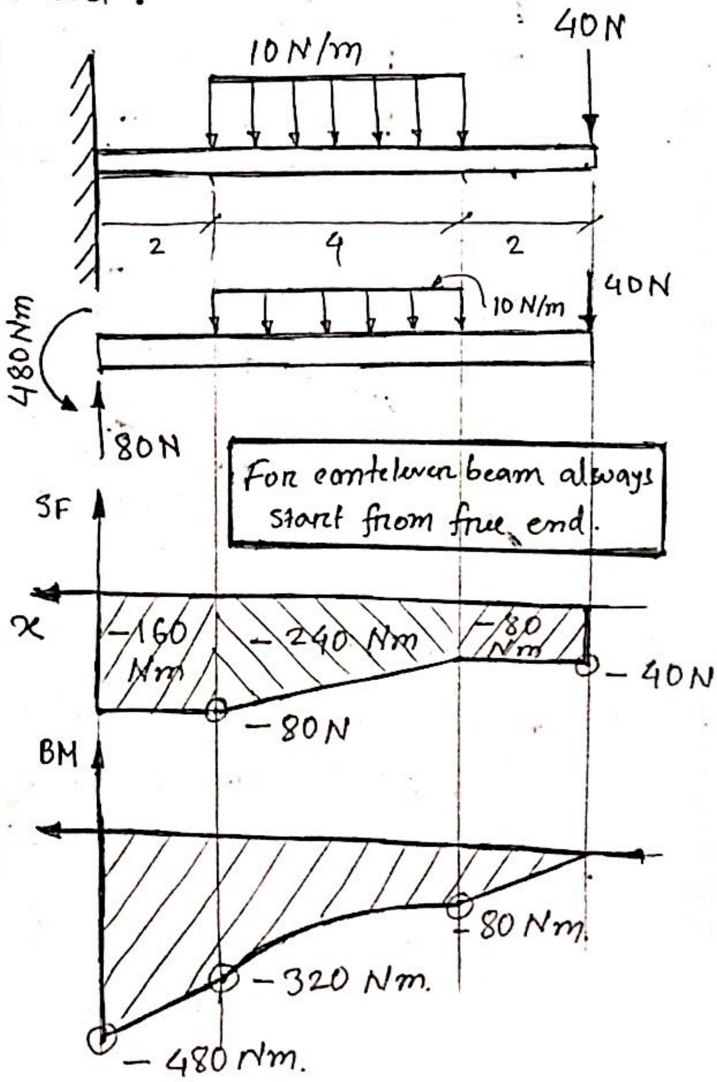
Maximum bending moment = 838.1 Nm (at $x = 2.757$ m).

Assignment: (2) Find the location and magnitude of maximum bending moment.



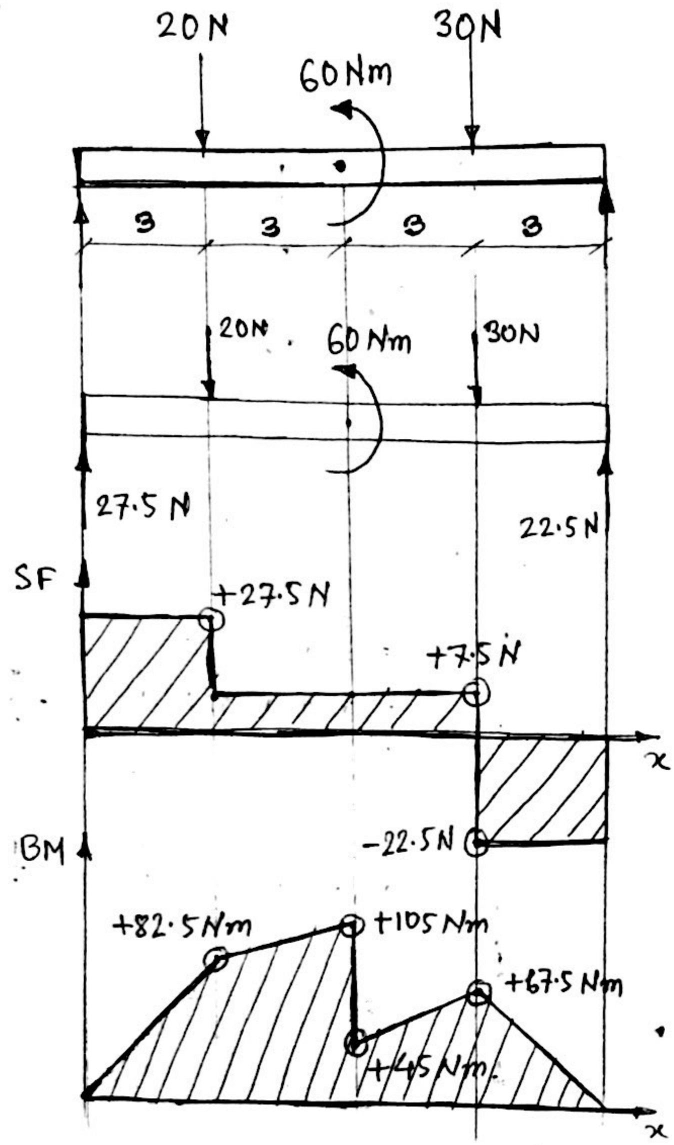
Moment diagram by area method :



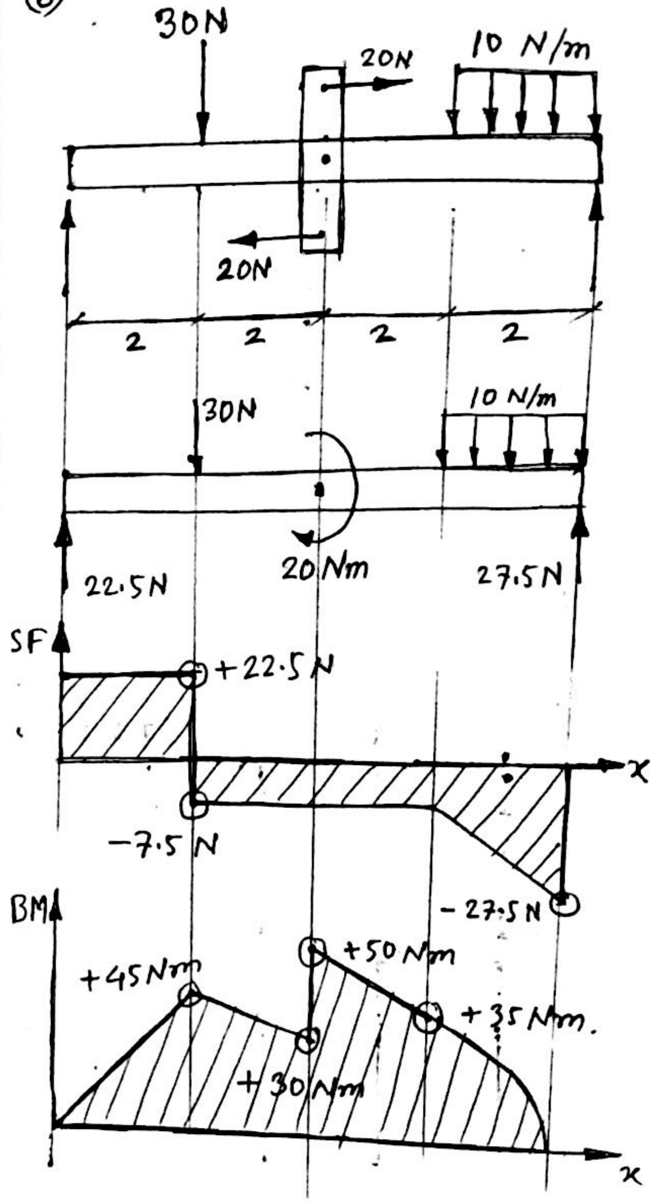


Problem:(14) Using area method draw SFD and BMD.

(a)



(b)



Conclusion:

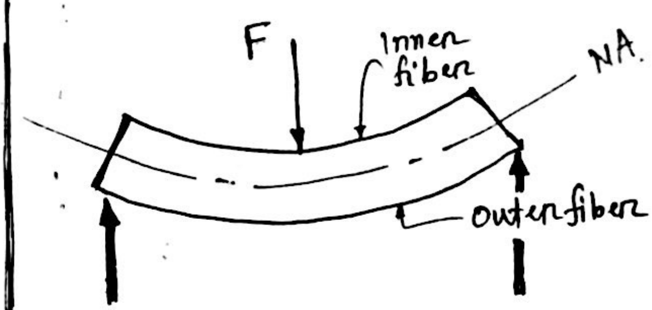
(a) External moment clockwise positive(+) and anticlockwise negetive(–).

(b) when an external moment is applied shear force diagram dosen't give equal area over and down x-axis.

Stress in beam: (Flexural stress)

→ when a beam is loaded it curves. due to the bending the outer fiber feels tension and inner fiber feels compression

→ Assumptions of flexural formula



ⓐ Pure bending (No torsion, No axial load)

ⓑ During bending crossection remain plane.

ⓒ Homogeneous material of beam

ⓓ Modulus of elasticity remain constant.

ⓔ Initially straight beam of uniform crossection.

ⓕ There is an axis of symmetry in the plane of bending.

ⓖ Pure transverse Load (Perpendicular to longitudinal axis)

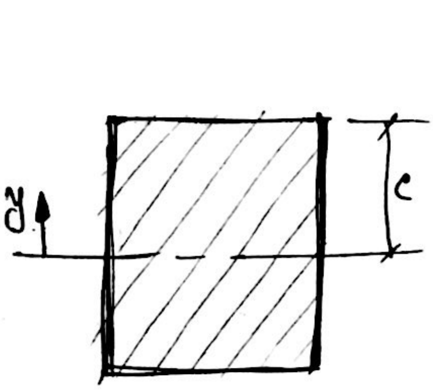
→ Flexural stress:, $\boxed{\sigma = \frac{My}{I}}$ $\boxed{\text{Proof : Hand note supplied}}$

Here, M = moment.

y = Arbitrary distance from neutral axis

I = Moment of inertia.

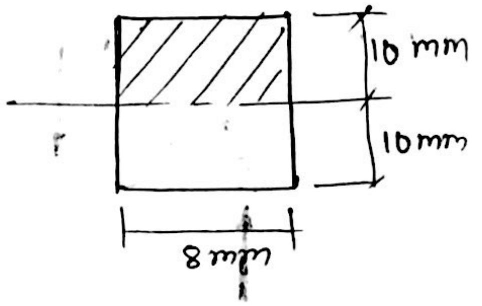
→ σ is maximum at outer most fiber.

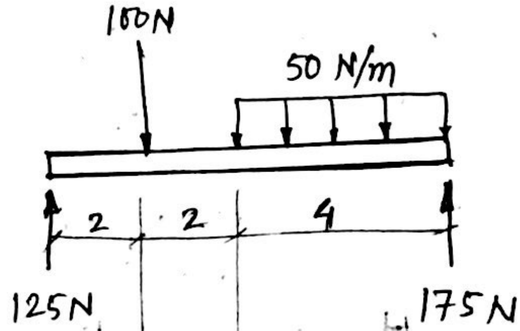


$$\therefore \boxed{(\sigma_{max}) = \frac{Mc}{I}}$$

→ σ is zero (0) on the neutral axis. ($y=0$).

Problem:(15) A beam is subjected to Load as shown in the figure. Find the maximum flexural stress in the beam.



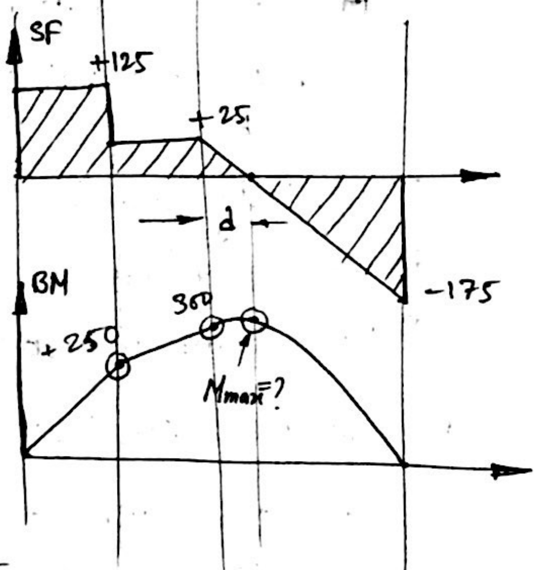


From SFD: $\frac{25}{d} = \frac{175}{4-d}$

$\Rightarrow d = 0.5$ m.

$\therefore M_{max} = 300 + \left(\frac{1}{2} \times 0.5 \times 25\right)$

$= 306.25$ Nm.

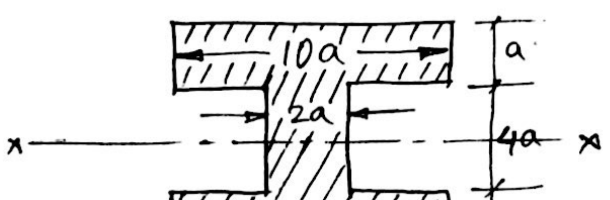


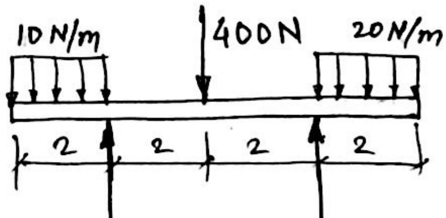
Maximum flexural stress

$$\sigma = \frac{Mc}{I} = \frac{306.25 \times 10}{\left\{\frac{1}{12} \times 8 \times 20^3\right\}}$$

$= 0.574$ MPa.

Problem:(16) Find the dimension of the beam, if the flexural strength of the beam material is 200 MPa.





in the beam.

From BMD: $M_{max} = 370$ Nm.

$\therefore \sigma = \dfrac{Mc}{I}$

Here, $c = 3a$ [symmetric section]

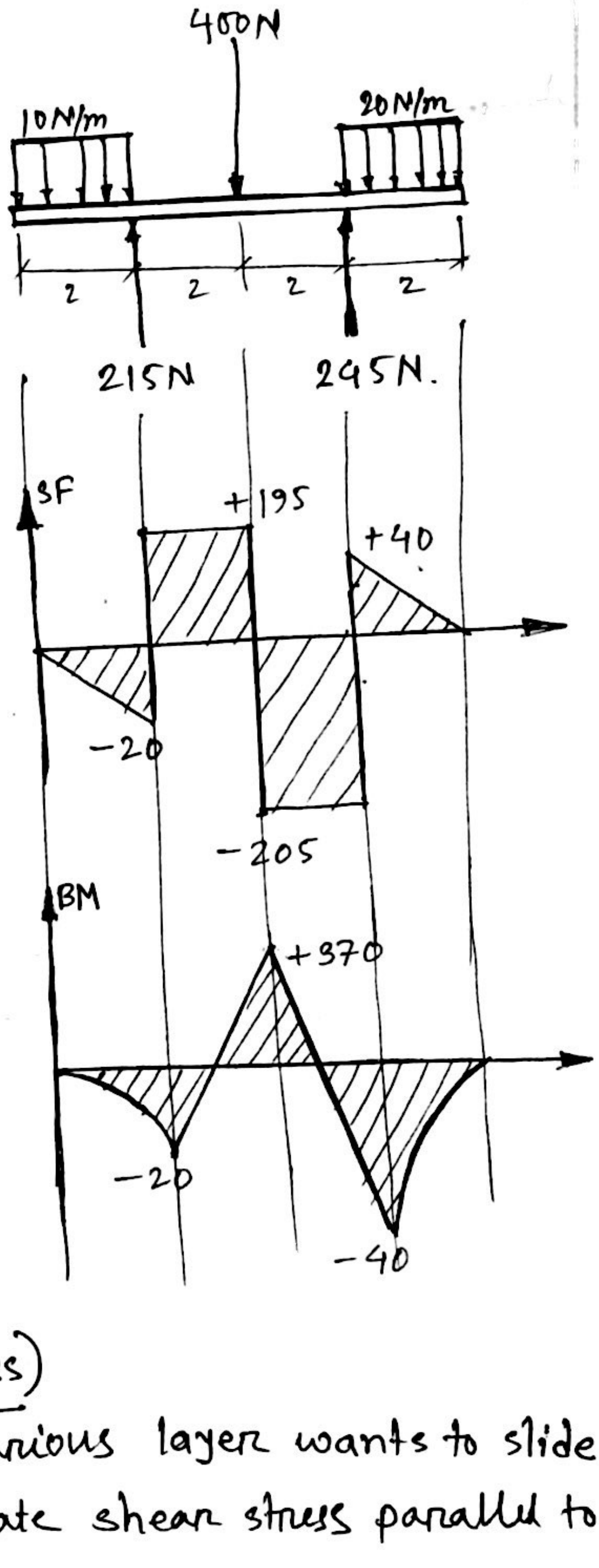
$$I = I_{xx} = \frac{1}{12} \cdot 10a(6a)^3 - 2\left[\frac{1}{12} \cdot 4a(4a)^3\right]$$

$$= 137.3 a^4 \quad mm^4.$$

$$\therefore \sigma = \frac{370 \times 1000 \times 3a}{137.3 a^4}$$

$$\Rightarrow 200 = \frac{8084.5}{a^3}$$

$$\Rightarrow a = 3.43 \text{ mm.}$$



Stress in beam: (shear stress)

→ when beam is bent the various layer wants to slide over one another. This create shear stress parallel to the bending plane.

→ Shear stress: $\boxed{\tau = \dfrac{VQ}{Ib}}$

V = shear force

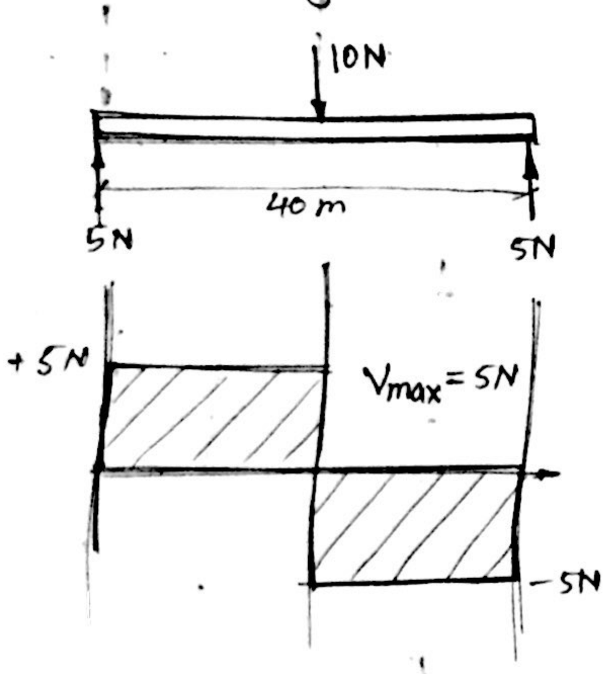
Q = first moment of area

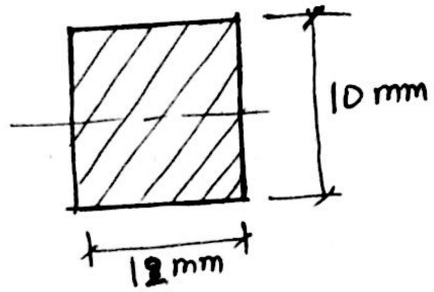
I = Second moment of area

b = width of beam

Proof: hand note supplied

→ Consider a beam of rectangular cross section as shown in figure:





Then, $V = 5\ N$.

$I = \frac{1}{12} \times 12 \times 10^3$

$= 1000\ mm^4$

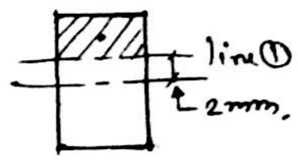
$b = 12\ mm$.

Now determine shear stress along neutral axis:

→ upper half slides: $Q = (5 \times 12) \times 2.5 = 150\ mm^3$

$$\therefore \tau = \frac{5 \times 150}{1000 \times 12} = 0.0625\ MPa$$

Again determine shear stress along line ①:



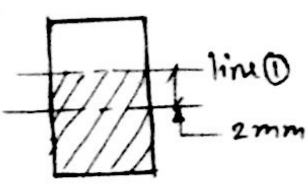
→ upper area slides: $Q = (3 \times 12) \times 3.5$

$= 126\ mm^3$

$$\therefore \tau = \frac{5 \times 126}{1000 \times 12} = 0.0525\ MPa.$$

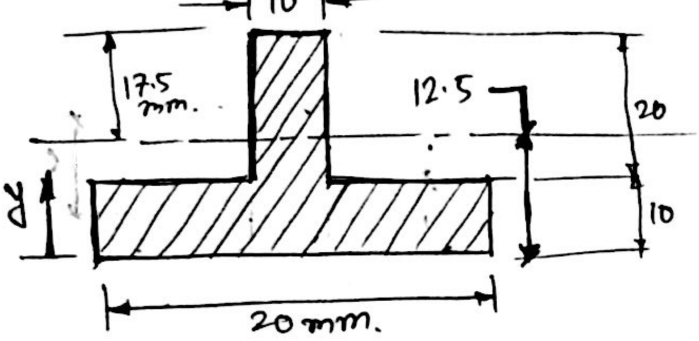
→ lower area slides: $Q = (7 \times 12) \times 1.5$

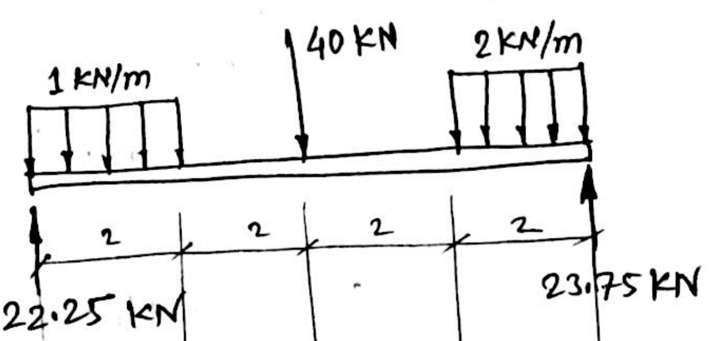
$= 126\ mm^3$

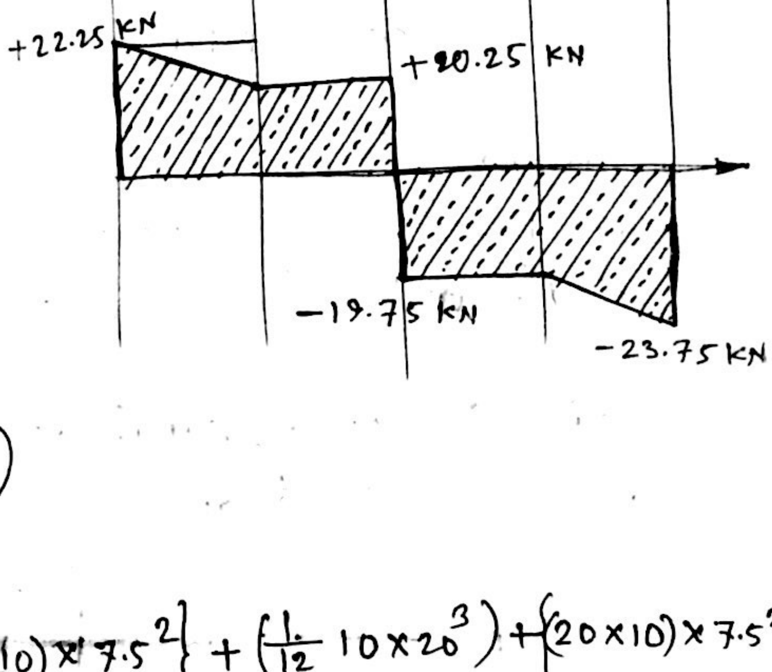


$$\therefore \tau = \frac{5 \times 126}{1000 \times 12} = 0.0525\ MPa.$$

Problem: (17) Find the maximum shearing stress along NA and line passes through $y = \frac{1}{4}c$ (ii) for the beam shown.







Here, $V = 19.75$ KN
$= 19750$ N.

$I =$ Neutral axis?

$$\therefore \bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A}$$

$$= \left(\frac{200 \times 5 + 200 \times 20}{200 + 200}\right)$$

$$= 12.5 \text{ mm.}$$

$$I = \left(\frac{1}{12}\, 20 \times 10^3\right) + \left\{(20 \times 10) \times 7.5^2\right\} + \left(\frac{1}{12}\, 10 \times 20^3\right) + \left\{(20 \times 10) \times 7.5^2\right\}$$

$$= 30833.3 \text{ mm}^4.$$

Along neutral axis: $Q = (17.5 \times 10) \times \left(\frac{17.5}{2}\right) = 1531.25.$

or, $Q = \left\{(20 \times 10) \times 7.5\right\} + \left\{(2.5 \times 10) \times 1.25\right\} = 1531.25$

$$\therefore \tau = \frac{VQ}{Ib} = \left\{\frac{19750 \times 1531.25}{30833.3 \times 10}\right\} = 98.1 \text{ MPa.}$$

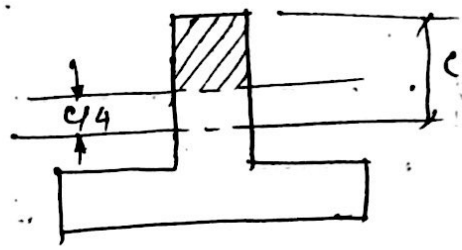
Along line ($y = \frac{1}{4}c$) $\Rightarrow$ $y = 4.375$ (From neutral axis).

$V = 19750\ N$

$Q = (13.125 \times 10) \times 10.9375 = 1435.55\ mm^3$

$I = 30833.3\ mm^4$

$b = 10\ mm.$



So, $\tau = \left(\dfrac{19750 \times 1435.55}{30833.3 \times 10}\right)$

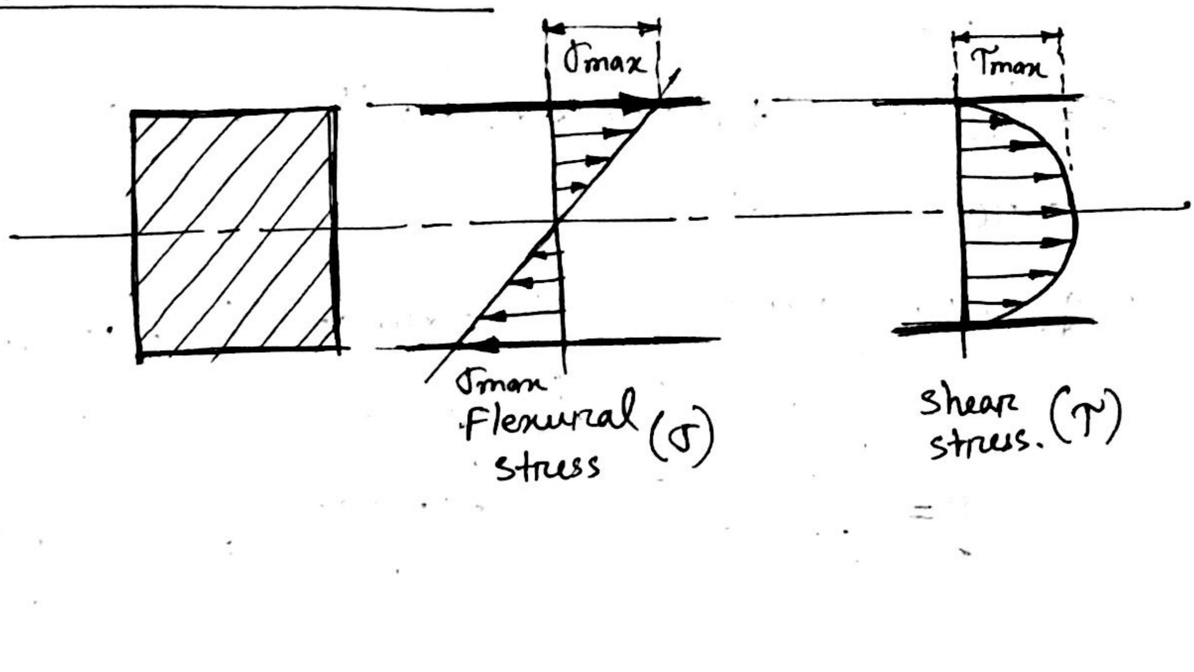
$= 91.95\ MPa.$

Conclusion:

→ shear stress is maximum at the neutral axis.

→ On outer fibers ($Q = 0$) shear stress is zero.

Distribution of flexural and shear stress over beam cross sectional area:

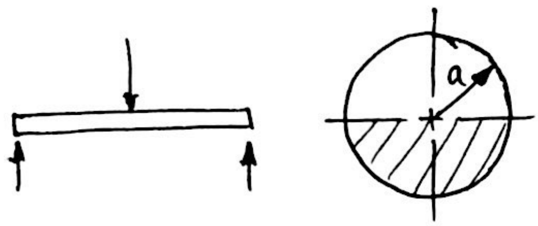


Flexural stress (σ)

Shear stress. (τ)

Problem: (18) Find the expression for Maximum flexural and shear stress for a beam with cross section of

(a) circular



* shear stress maximum on the line passing through center

$$Q = \left(\pi \frac{r^2}{2}\right)\cdot\left(\frac{4r}{3\pi}\right) = \frac{2}{3} r^3$$

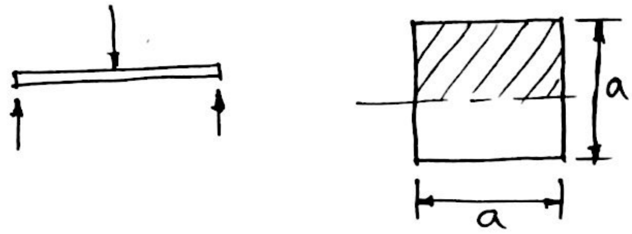
$$I = \left(\pi \frac{r^4}{4}\right) \; ; \; b = 2r$$

$$\therefore \tau = \frac{4V}{3A} = \frac{4}{3}\left(\frac{V}{A}\right).$$

* Flexural stress maximum on the top/Bottom fiber.

$$\sigma = \frac{Mc}{I} = \frac{4}{r}\left(\frac{M}{A}\right)$$

(b) square.



* shear stress maximum on the line passing through center.

$$Q = \left(a \cdot \frac{a}{2}\right)\cdot\left(\frac{a}{4}\right) = \frac{a^3}{8}$$

$$I = \frac{1}{12} a \cdot a^3 = \frac{1}{12} a^4 \; ; \; b = a$$

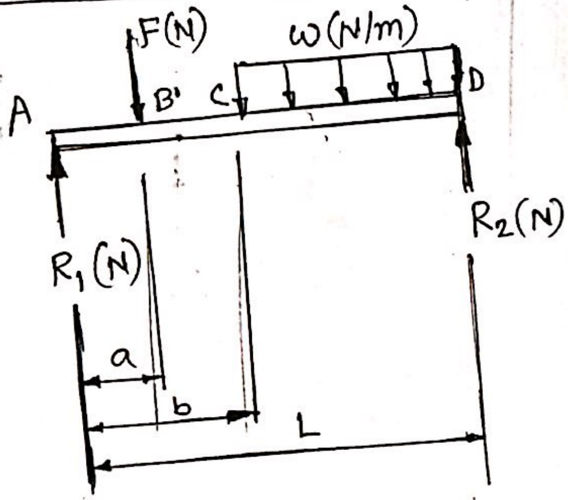
$$\therefore \tau = \frac{3V}{2A} = \frac{3}{2}\left(\frac{V}{A}\right)$$

* Flexural stress maximum on the outer/inner fiber.

$$\sigma = \frac{Mc}{I} = \frac{6}{a}\left(\frac{M}{A}\right).$$

Generalized moment equation:

- Single equation with Justification mark $< >$.
- Justification mark indicates that "if the value inside is negative it is to be neglected, otherwise treat it as other regular term".
- Every distributed load must be continuous upto the last point. Add nessary loads to do so.



For AB:

$$M = R_1 x \quad \text{------------} \; (\text{i})$$

For BC:

$$M = R_1 x - F(x-a) \quad \text{-----} \; (\text{ii})$$

For CD:

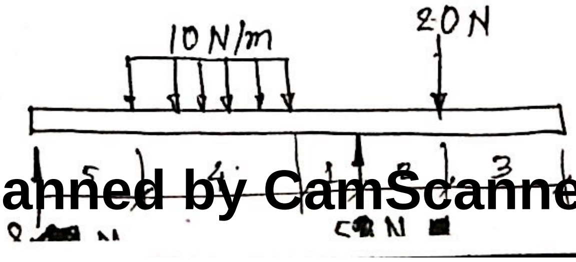
$$M = R_1 x - F(x-a) - \frac{w}{2}(x-c)^2 \quad \text{------} \; (\text{iii})$$

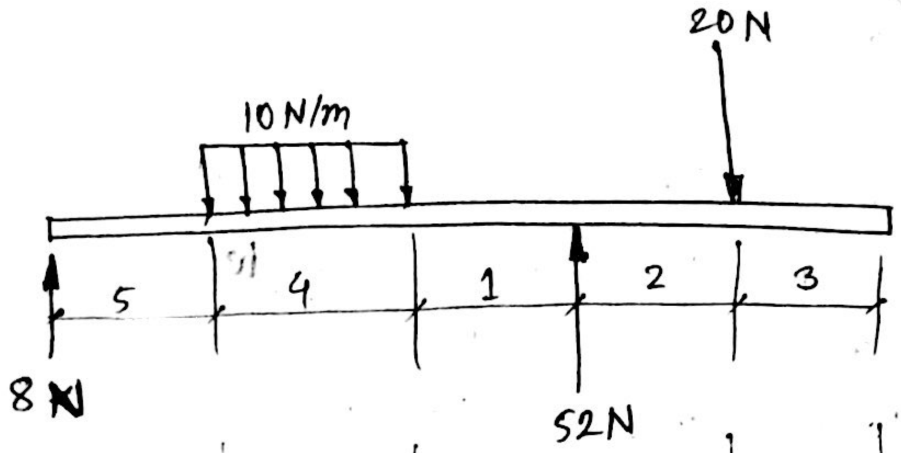
equation (iii) also hold all the term. So it can be used as generalized equation like:

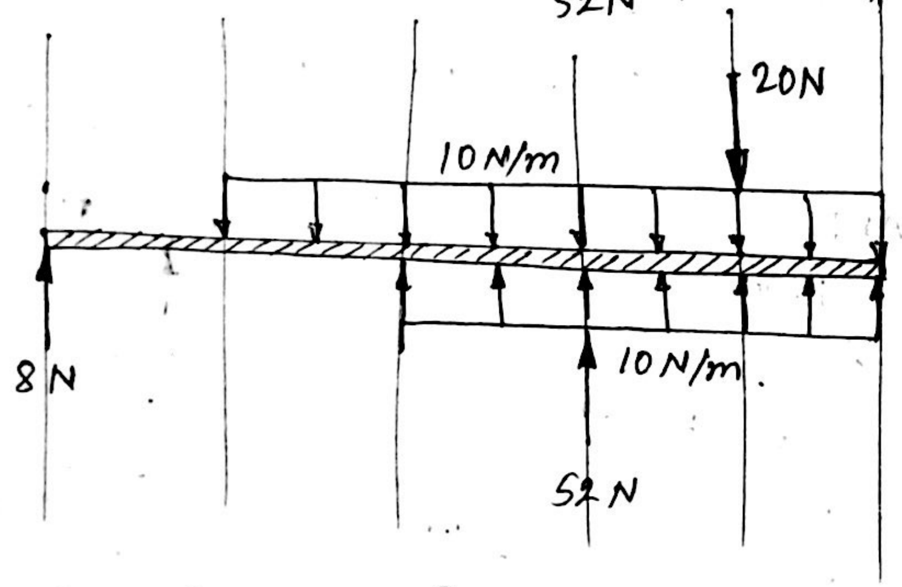
$$M = R_1 x - F\langle x-a \rangle - \frac{w}{2}\langle x-c \rangle^2.$$

- So moment equation for right most segment with justification mark is the generalized moment equatio

Problem: (19) Find the generalized moment equation for the beam shown:







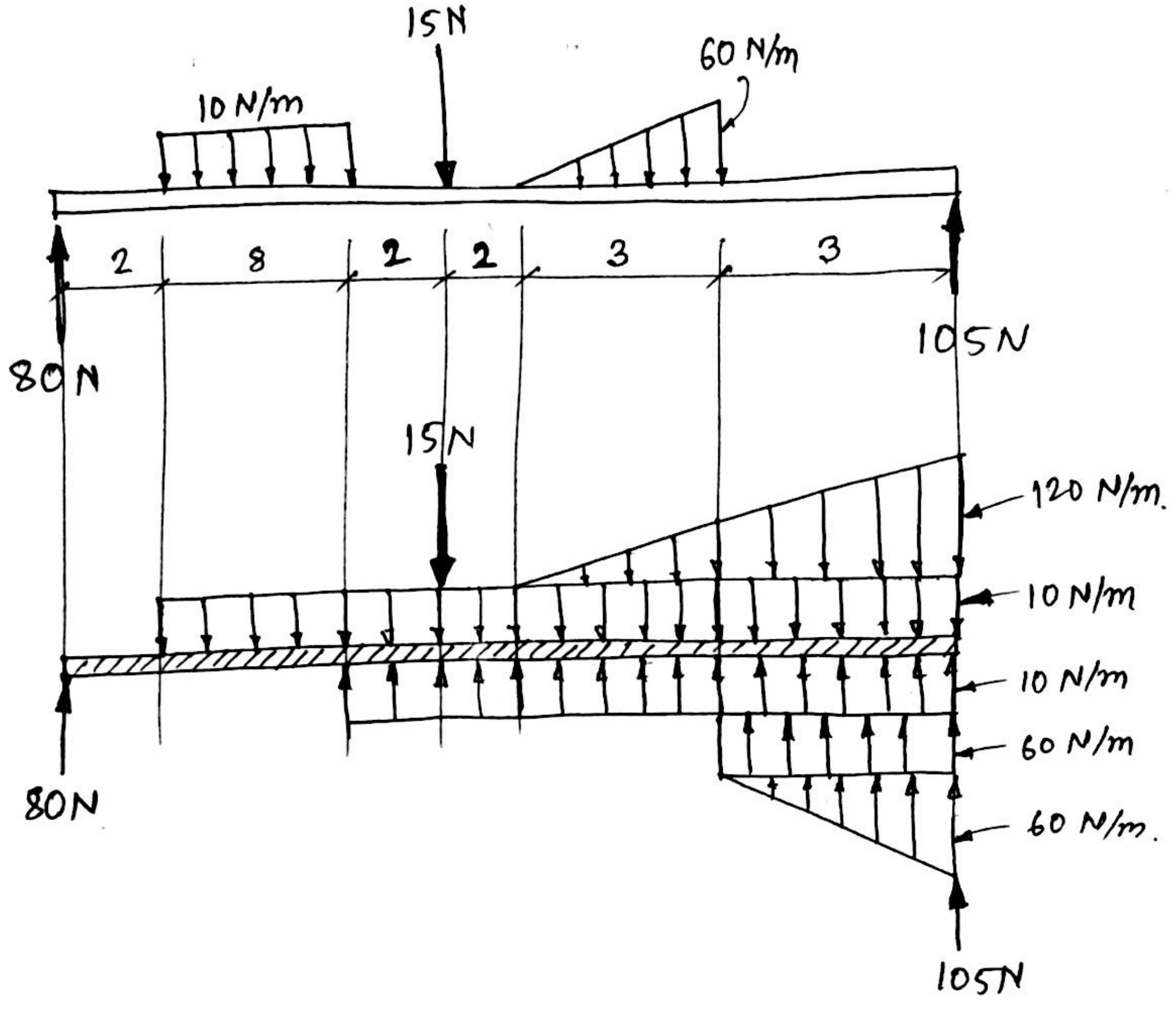
So, $M = 8x - \frac{10}{2}\langle x-5\rangle^2 + \frac{10}{2}\langle x-9\rangle^2 + 52\langle x-10\rangle$
$- 20\langle x-12\rangle$.

→ So, For point loading, $M = F\langle x-a\rangle$

→ For uniform distributed loading, $M = \frac{\omega}{2}\langle x-a\rangle^2$

→ For triangular distributed loading, $M = \frac{\omega}{6(L-a)}\langle x-a\rangle^3$.

Problem: (20) Find generalized moment equation.



Generalized moment equation:

$$M = 80x - \frac{10}{2}\langle x-2\rangle^2 + \frac{10}{2}\langle x-10\rangle^2 - 15\langle x-12\rangle$$

$$- \frac{120}{6\times(20-14)}\langle x-14\rangle^3 + \frac{60}{2}\langle x-17\rangle^3$$

$$+ \frac{60}{6(20-17)}\langle x-17\rangle^3.$$

$$\Rightarrow M = 80x - 5\langle x-2\rangle^2 + 5\langle x-10\rangle^2 - 15\langle x-12\rangle - \frac{10}{3}\langle x-14\rangle^3 + 30\langle x-17\rangle^3 + \frac{10}{3}\langle x-17\rangle^3.$$

Beam deflection: (Double integration method)

→ According to small deflection theory:

$$\boxed{EI\left(\frac{d^2y}{dx^2}\right) = M}$$

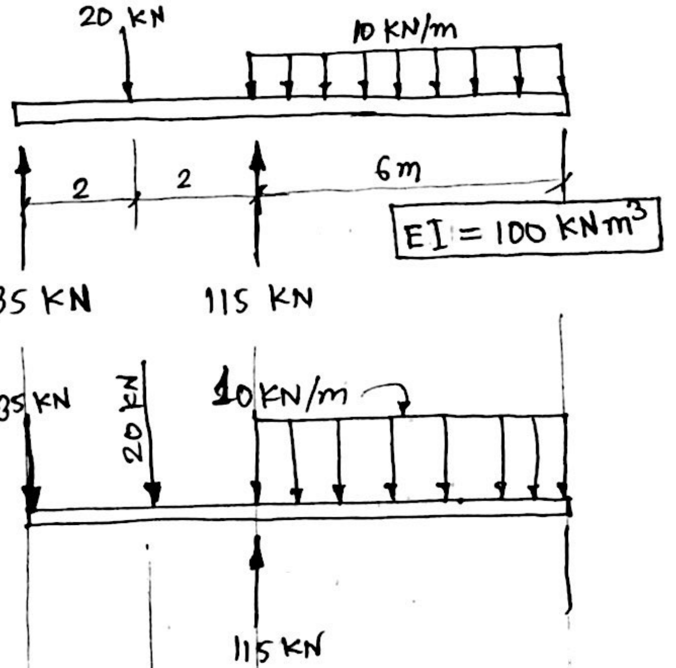
Proof: handnote supplied

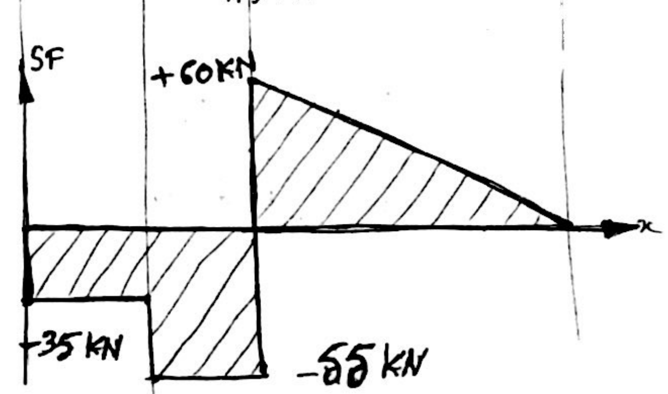
→ Where, E = Modulus of elasticity, I = Moment of Inertia } EI = Flexural rigidity.

y = Equation of elastic curve.

M = Moment equation.

Problem: (21) Find the deflection of the beam at mid span.





Generalized moment equation:

$$M = -35x - 20\langle x-2\rangle + 115\langle x-4\rangle - 5\langle x-4\rangle^2 \quad \boxed{KN\,m}$$

$$\int M\,dx = -\frac{35}{2}x^2 - 10\langle x-2\rangle^2 + \frac{115}{2}\langle x-4\rangle^2 - \frac{5}{3}\langle x-4\rangle^3 + C_1 \quad \boxed{KN\,m^2}$$

$$\iint M\,dx\,dx = -\frac{35}{6}x^3 - \frac{10}{3}\langle x-2\rangle^3 + \frac{115}{6}\langle x-4\rangle^3 - \frac{5}{12}\langle x-4\rangle^4 + C_1x + C_2 \quad \boxed{KN\,m^3}$$

$$\Rightarrow EIy = -\frac{35}{6}x^3 - \frac{10}{3}\langle x-2\rangle^3 + \frac{115}{6}\langle x-4\rangle^3 - \frac{5}{12}\langle x-4\rangle^4 + C_1x + C_2$$

when, $x = 0$, $y = 0$; → $C_2 = 0$

Again, $x = 4,\ y = 0;\ \rightarrow 0 = -\frac{35}{6}(4)^3 - \frac{10}{3}(2)^3 + 4C_1$

$$\Rightarrow C_1 = (373.3 + 26.7)/4$$

$$\Rightarrow C_1 = 100.$$

So, $EIy = -\frac{35}{6}x^3 - \frac{10}{3}\langle x-2\rangle^3 + \frac{115}{6}\langle x-4\rangle^3 - \frac{5}{12}\langle x-4\rangle^4 + 100x$

$$\Rightarrow \boxed{y = -\frac{35}{600}x^3 - \frac{10}{300}\langle x-2\rangle^3 + \frac{115}{600}\langle x-4\rangle^3 - \frac{5}{1200}\langle x-4\rangle^4 + x}$$

Deflection at mid span: $(x = 10/2 = 5\ m)$

$$\delta_{x=5} = -\frac{35}{600}(125) - \frac{10}{300}(27) + \frac{115}{600}(1) - \frac{5}{1200}(1) + 5$$

$$= -7.29 - 0.9 + 0.19 - 0.004 + 5 = -3.194\ m.$$

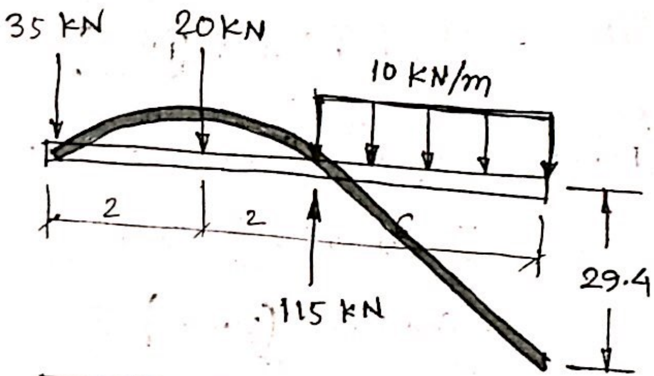
Ans.

<u>Understanding elastic curve:</u>

To find location and value of maximum deflection -

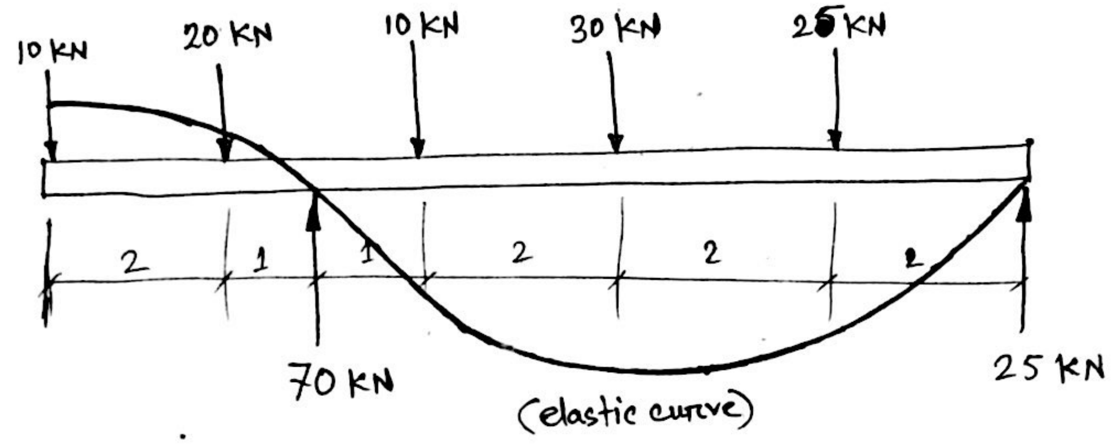
(i) Set $\int M\,dx = 0$ and get where the deflection is maximum.

(ii) Put the value of x in $y = \frac{1}{EI}\iint M\,dx\,dx$.



x	y
0	0
1	+ 0.94
2	+ 1.53
3	+ 1.39
4	0
7	− 12.3

Problem: (22) Find the maximum deflection and its location:



$$M = -10x - 20\langle x-2\rangle + 70\langle x-3\rangle - 10\langle x-4\rangle - 30\langle x-6\rangle - 25\langle x-8\rangle$$

$$\Rightarrow \int M\,dx = -5x^2 - 10\langle x-2\rangle^2 + 35\langle x-3\rangle^2 - 5\langle x-4\rangle^2 - 15\langle x-6\rangle^2 - 12.5\langle x-8\rangle^2 + \ldots$$

$$\Rightarrow \iint M\,dx\,dx = -\frac{5}{3}x^3 - \frac{10}{3}\langle x-2\rangle^3 + \frac{35}{3}\langle x-3\rangle^3 - \frac{5}{3}\langle x-4\rangle^3 - 5\langle x-6\rangle^3 - 4.17\langle x-8\rangle^3 + C_1x + C_2$$

$$\Rightarrow EIy = -(1.67)x^3 - 3.33\langle x-2\rangle^3 + 11.67\langle x-3\rangle^3 - 1.67\langle x-4\rangle^3 - 5\langle x-6\rangle^3 - 4.167\langle x-8\rangle^3 + C_1x + C_2.$$

when, $x = 3$; $y = 0 \rightarrow 3C_1 + C_2 = 48.33$

$x = 10$; $y = 0 \rightarrow 10C_1 + C_2 = 86.206$

$C_1 = 5.41$, $C_2 = 32.1$.

$$\therefore y = \frac{1}{EI}\left[-1.67x^3 - 3.33\langle x-2\rangle^3 + 11.67\langle x-3\rangle^3 - 1.67\langle x-4\rangle^3 - 5\langle x-6\rangle^3 - 4.167\langle x-8\rangle^3 + 5.41x + 32.1\right]$$

x	y
0	32.1
1	35.8
2	29.5
3	0

x	y
4	−68.1
5	−147.8
6	−207.5
7	−227.2

x	y
8	−187.1
9	−106.0
10	0

For maximum deflection; $\frac{dy}{dx} = 0$ $\boxed{6 < x < 8}$ Assumed.

$$\therefore -5x^2 - 10(x-2)^2 + 35(x-3)^2 - 5(x-4)^2 - 15(x-6)^2 - 5.41 = 0$$

$$\Rightarrow 50x - 350.41 = 0$$

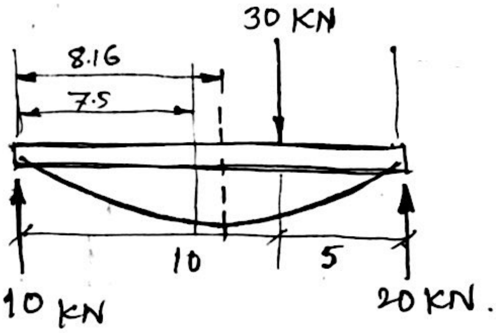
$$\Rightarrow x = 7.0082 \text{ m.}$$

$$\approx 7 \text{ m.} \longrightarrow \boxed{6 < x < 8 \text{ (true)}}$$

So, maximum deflection:

$$\delta_{max} = -222.3 \text{ m.}$$ $\boxed{\text{putting } x=7 \text{ in expression of } y}$

Problem: (23): Find the location and value of maximum deflection of the beam.



Solution: $M = 10x - 30\langle x-10\rangle$

$$\therefore \int M\,dx = 5x^2 - 15\langle x-10\rangle^2 + C_1$$

$$\iint M\,dx\,dx = \frac{5}{3}x^3 - 5\langle x-10\rangle^3 + C_1 x + C_2$$

when, $x=0,\ y=0 \longrightarrow C_2 = 0$

$x=15,\ y=0 \longrightarrow C_1 = -333.33$.

$$\therefore EIy = \frac{5}{3}x^3 - 5\langle x-10\rangle^3 - 333.33x.$$

$$\Rightarrow y = \frac{5}{3}x^3 - 5\langle x-10\rangle^3 - 333.33x$$ $\boxed{EI = 1 \text{ kNmm}^3}$

Assume: maximum deflection occurs in $0 < x < 10$.

$\therefore \frac{dy}{dx} = 0$; $5x^2 - 333.33 = 0$; $x = 8.16$ $\boxed{\text{correct assumption}}$

Assume: maximum deflection occurs $10 < x < 15$.

$\therefore \frac{dy}{dx} = 0$; $5x^2 - 15(x-10)^2 - 333.33 = 0$; $x = 8.55, 21.45$ $\boxed{\text{Wrong assumption}}$

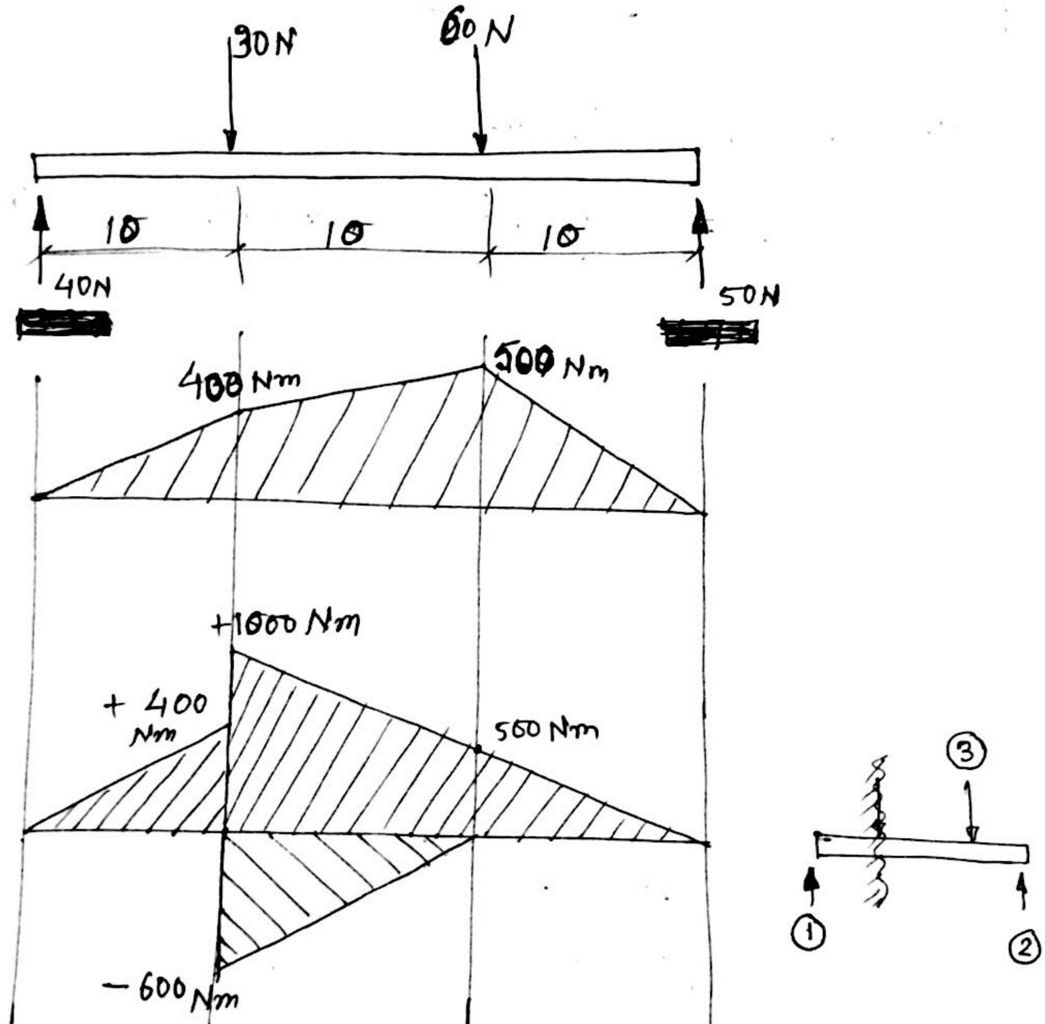
Now maximum deflection:

$$\delta_{max} = \frac{5}{3}(8.16)^3 - 333.33 \times 8.16 = -1814.4 \text{ m.}$$

Beam deflection (Moment method)

- Deals with area under moment diagram.
- Generally moment diagram is drawn in parts for easier area calculation.
- Moment diagram by parts consist several cantelever beam.

Problem: (24) Draw the moment diagram of the beam in both moment by parts and area methode.



Area under segment.	$+2000\ Nm^2$	$+4500\ Nm^2$	$+2500\ Nm^2$	upper diagram
	$+2000\ Nm^2$	$+7500-3000 = 4500\ Nm^2$	$+2500\ Nm^2$	lower diagram

→ Infinity no of combination for moment diagram by parts are possible. Each logical one is correct.

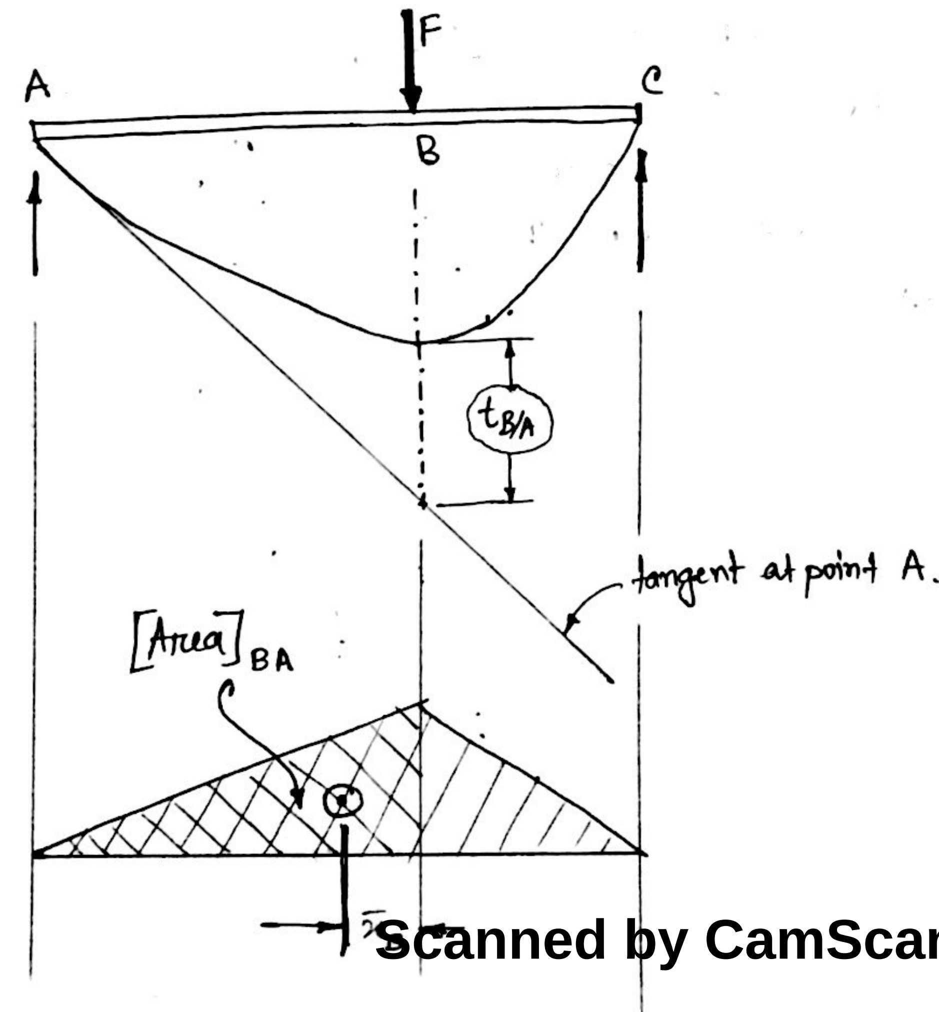
→ The theorem is expressed as:

$$t_{B/A} = \frac{1}{EI}\left[\text{Area}\right]_{BA} \cdot \overline{x}_B$$

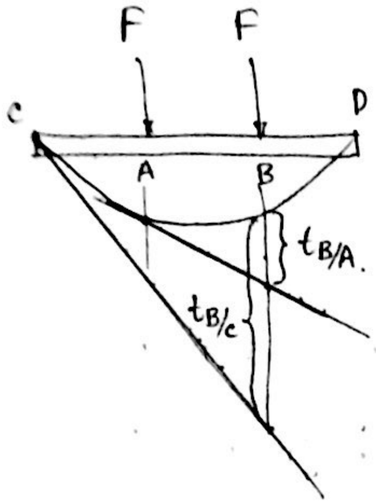
Here, $t_{B/A}$ → tangential deviation of B with respect to A.

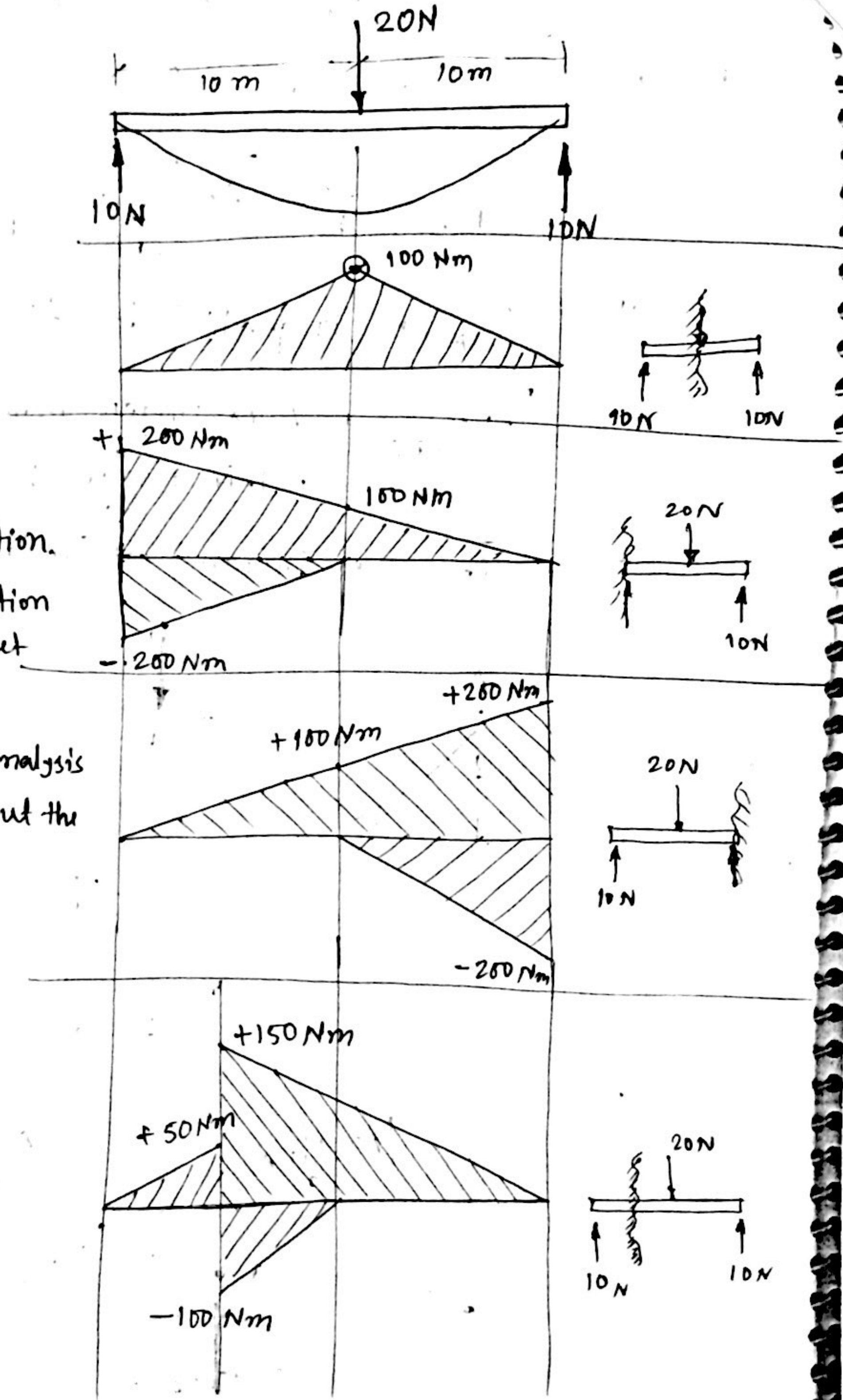
$[\text{Area}]_{BA}$ → Area under moment diagram (Sum) within range B to A.

$\overline{x}_B$ → Distance of $(\text{area})_{BA}$ from B.



More insight to moment diagram by parts:

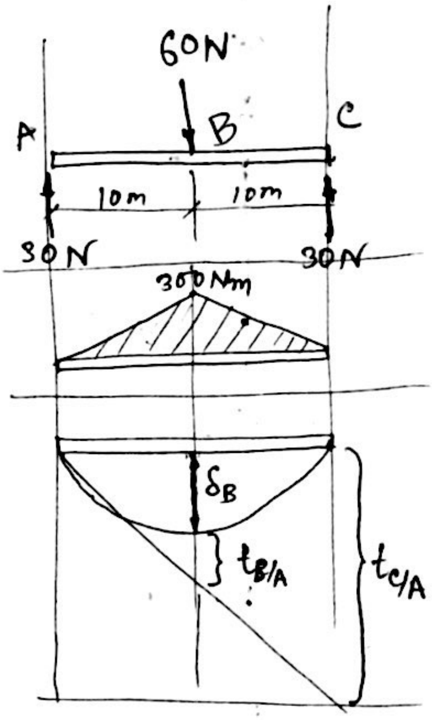




- Do not give the deflection.
- Provide tangential deviation of one point with respect to another point.
- further geometrical analysis is required to find out the deviation.

Problem: (25) Find the deflection at $x = 10$; (midspan) Given that $EI = 10^6 \ Nm^3$.

Solution:



$$t_{B/A} = \left(\frac{1}{10^6}\right)\left[\left(\frac{1}{2} \times 300 \times 10\right) \times \frac{10}{3}\right]$$

$$= 0.005 \ m.$$

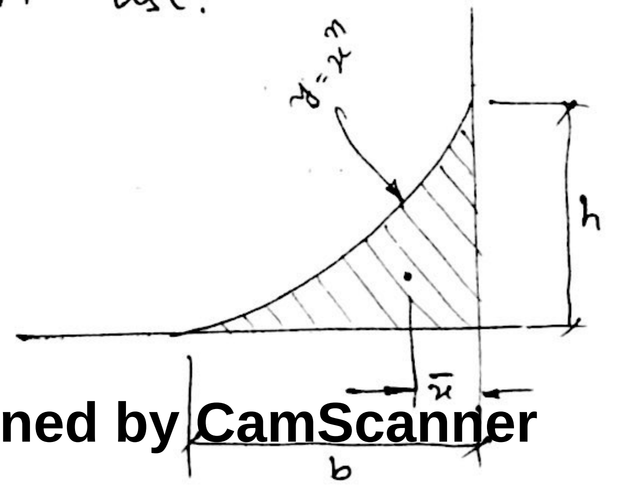
$$t_{C/A} = \left(\frac{1}{10^6}\right)\left[\left(\frac{1}{2} \times 300 \times 10\right) \times \left(10 + \frac{10}{3}\right) + \left(\frac{1}{2} \times 300 \times 10\right) \times \frac{20}{3}\right]$$

$$= 0.03 \ m.$$

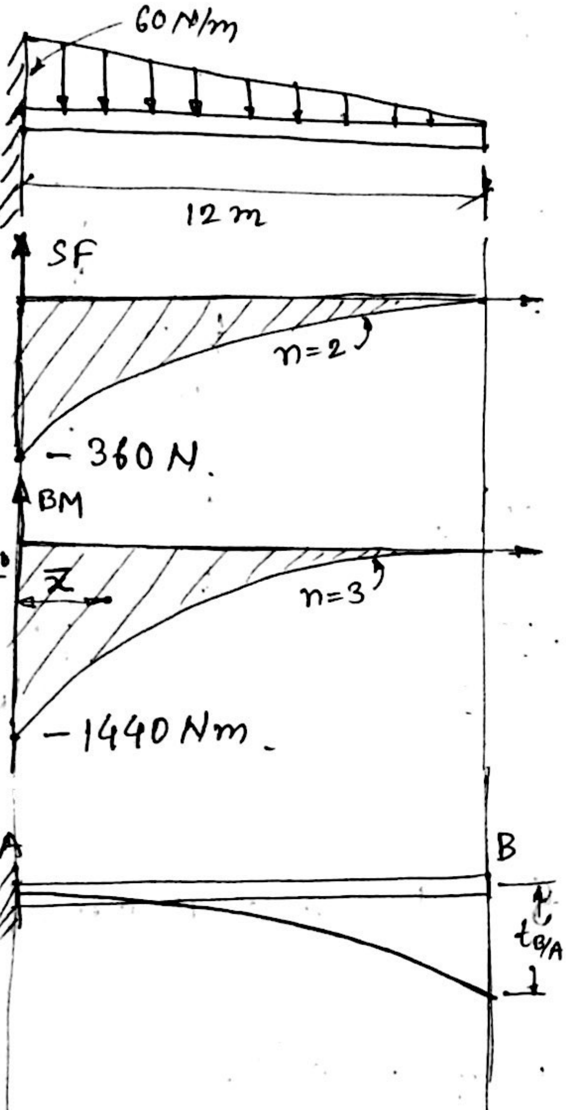
Now, $\frac{t_{C/A}}{20} = \frac{t_{B/A} + \delta_B}{10}$; $\delta_B = 0.01 \ m = 10 \ mm$. Ans.

- Very easy way to find beam deflection.
- Need to know different area and its center of gravity,
- For equation of order "n" use:

$A = \left(\frac{1}{n+1}\right) bh$
$\bar{x} = \left(\frac{1}{n+2}\right) b$



Problem: (26) Find the deflection at the free end $EI = 10^6\ Nm^2$



$$A = \frac{12 \times 1440}{3+1} = 4320$$

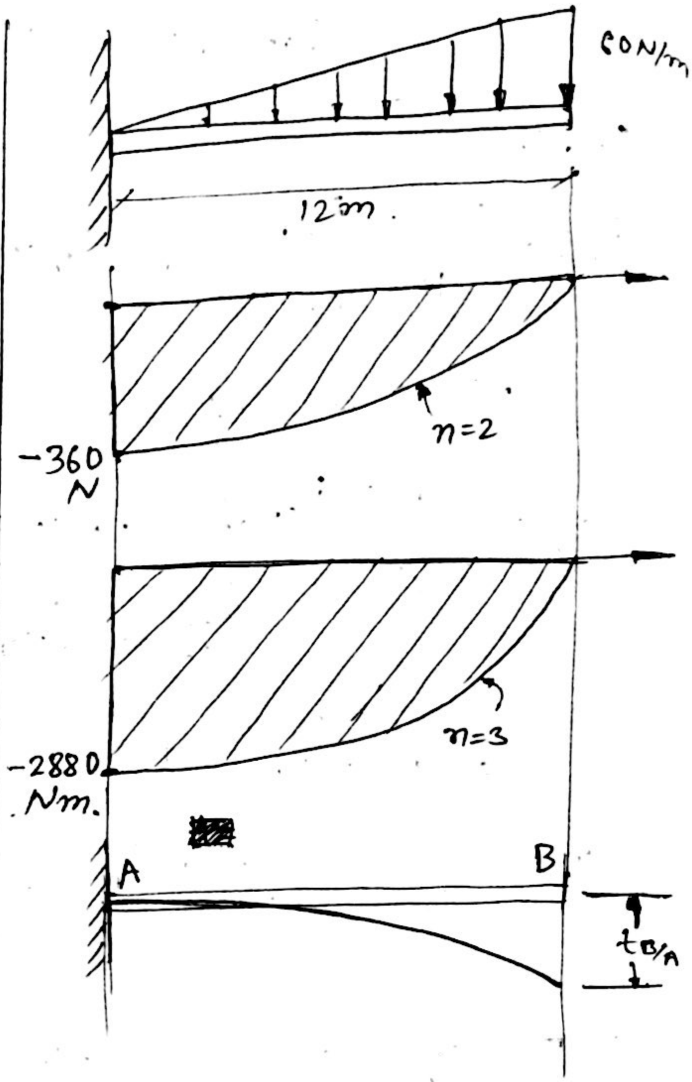
$$\bar{x} = \frac{12}{3+2} = 2.4$$

$$\delta_B = t_{B/A}$$

$$= \frac{1}{10^6}\left[4320 \times (12 - 2.4)\right]$$

$$= 0.0414\ m$$

$$= 41.4\ mm \quad \text{(Ans)}$$



$$t_{B/A} = \left(\frac{1}{10^6}\right)\left[(2880 \times 12 \times 6) - \left(\frac{1}{4}\, 2880 \times 12 \times \frac{12}{5}\right)\right]$$

$$= 0.1866\ m$$

$$= 186.6\ mm \quad \text{(Ans)}$$

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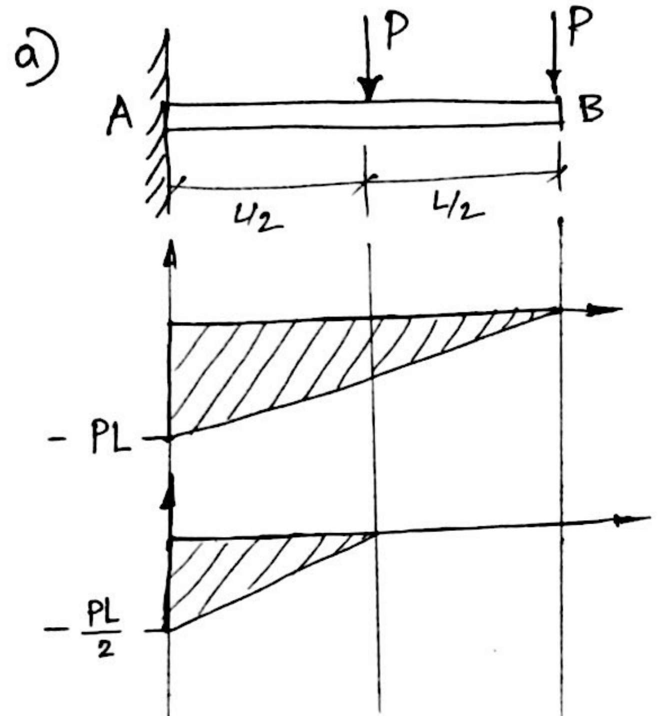
Problem: (27) Find the expression for deflection at free end.

a)



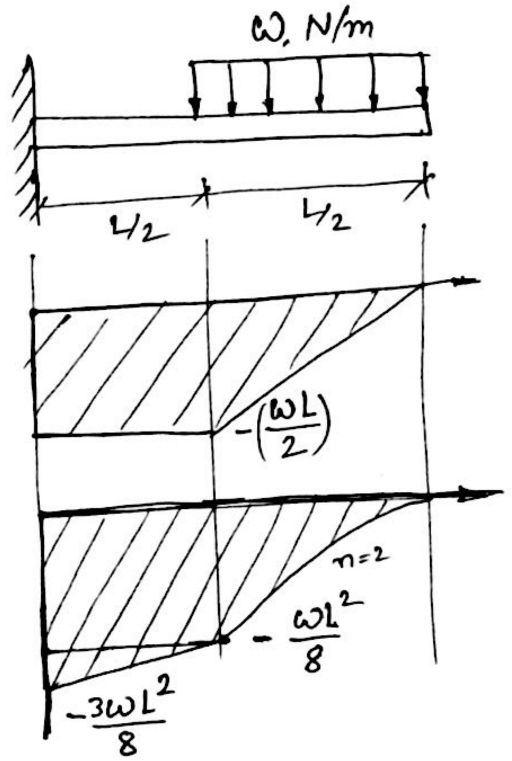
$$t_{B/A} = -\frac{1}{EI}\left[\frac{PL^2}{2} \times \frac{2L}{3} + \frac{PL^2}{8} \times \left(\frac{L}{2} + \frac{2L}{6}\right)\right]$$

$$= \frac{PL^3}{EI}\left[\frac{1}{3} + \frac{5}{48}\right] = \frac{-21 \cdot PL^3}{48EI}$$

As, $t_{B/A}$ is downward as all area are negetive:

$$\therefore \delta_B = -\left(\frac{21PL^3}{48EI}\right)$$

b)

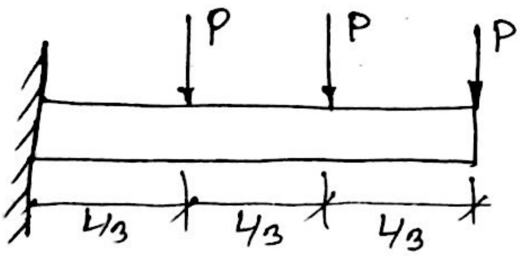


$$t_{B/A} = -\frac{1}{EI}\left[\frac{1}{3}\cdot\frac{\omega L^2}{8}\cdot\frac{L}{2}\left(\frac{L}{2} - \frac{1}{4}\frac{L}{2}\right) + \frac{\omega L^2}{8} \times \frac{L}{2} \times \left(\frac{L}{2} + \frac{L}{4}\right) + \frac{1}{2}\,\frac{3\omega L^2}{8} \times \frac{L}{2} \times \left(\frac{L}{2} + \frac{2}{3} \times \frac{L}{2}\right)\right]$$

$$= -\frac{\omega L^4}{EI}\left[\frac{1}{128} + \frac{3}{64} + \frac{5}{64}\right]$$

$$= -\left(\frac{17\,\omega L^4}{128\,EI}\right)$$

Assignment: Free end deflection



Torsional stress (Torsional shear stress)

→ When a body is subjected to torsion every cross section experience a shear stress.

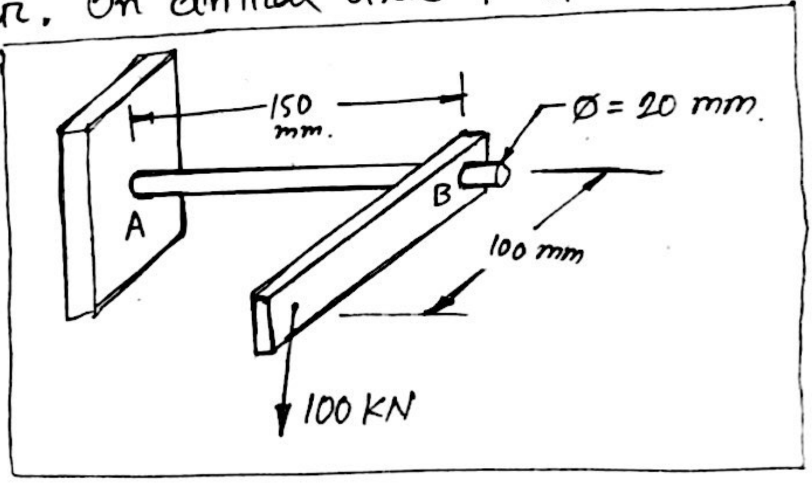
→ The formula of shear stress developed due to torsion:

$$\boxed{\tau = \frac{T\rho}{J}}$$

T = applied torque
ρ = radial distance from center of cross section.
J = Polar moment of inertia
$= I_x + I_y$. (whole area).

Proof: hand note supplied

→ τ is maximum at the most outer fiber. On central axis $\tau = 0$.



Problem: 28 Find the torsional shear stress developed in rod AB, in the plane where $\rho = \frac{1}{2}(r)$ and $\rho = r$.

Solution: Here, $T = (100 \times 10^3 \times 100) = 10^7$ Nmm.

$$J = \pi\left(\frac{20^4}{32}\right) = 15708 \text{ mm}^4.$$

at, $\rho = r/2$; $\tau = \frac{10^7 \times 5}{15708} = 3183.1$ MPa

at $\rho = r$; $\tau = \frac{10^7 \times 10}{15708} = 6366.2$ MPa

Torsional deformation:

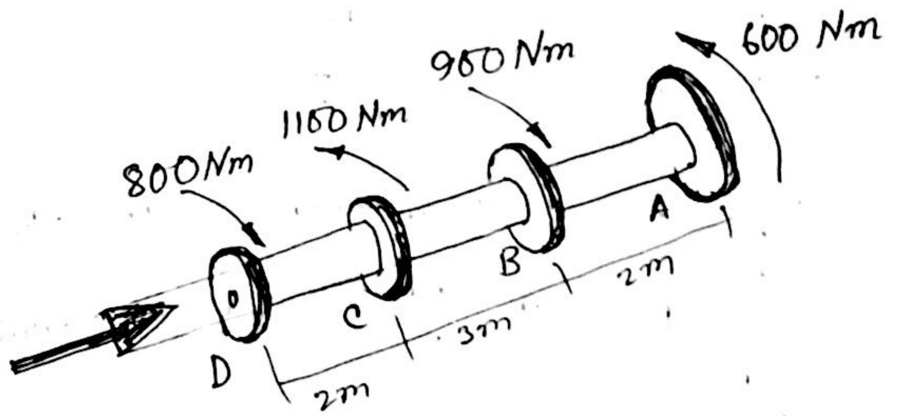
→ Modulus of rigidity: $G = \dfrac{\text{shear stress}}{\text{shear strain}}$

→ Shear strain / deformation: $\boxed{\theta = \dfrac{TL}{GJ}}$ Proof — hand note

Problem:(29) For problem:28 find the angular deformation of point B respect to point A. $\boxed{G = 80\ GPa}$

$$\theta = \left(\frac{10^7 \times 150}{80 \times 10^3 \times 15708}\right) = 1.19 \text{ rad}$$

Problem:(30) An aluminium shaft with a diameter of 50 mm is loaded by torques applied to it as shown in the figure. Using $G = 28$ GPa determine the angle of twist of gear D relative to gear A.



Solution: Consider clockwise positive (looking from D).

$$\theta_{D/A} = \theta_{D/C} + \theta_{C/B} + \theta_{B/A}.$$

$J = \frac{\pi d^4}{32} = 61359.32 \text{ mm}^4$; $G = 28 \times 10^3$ MPa.

$$\therefore \theta_{D/A} = \left(\frac{TL}{GJ}\right)_{D-C} + \left(\frac{TL}{GJ}\right)_{C-B} + \left(\frac{TL}{GJ}\right)_{B-A}$$

$$= \frac{800 \times 10^3 \times 2000}{GJ} + \frac{(-300) \times 10^3 \times 3000}{GJ} + \frac{600 \times 10^3 \times 2000}{GJ}$$

$$= 0.0931 - 0.0524 + 0.0695$$

$$= 0.1102 \text{ rad } (6.31^\circ) \text{ (clockwise)}.$$

Problem:(31) A steel rod of diameter 50 mm has shear strength of 100 MPa. Find the amount of maximum power that can be transmited through the rod at 120 rpm.

Solution: let maximum torque that can be applied on the shaft is 'T' Nmm.

$$\therefore \tau = \frac{T \times 50}{\pi \frac{(50)^4}{32}} \;;\; T = 1227187.5 \text{ Nmm} = 1227.2 \text{ Nm}.$$

power, $P = T\omega$

$$= 1227.2 \times \left(\frac{120 \times 2\pi}{60}\right) = 15421 \text{ W}$$

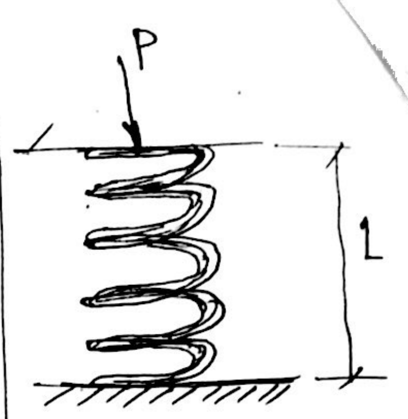
$$= 15.421 \text{ KW}$$

(Ans)

Spring: (helical spring)

→ helix of a [redacted] wire.

→ Every cross-section is experienced a direct shear stress and a torsional shear stress.



→ Total shear:

$$\tau_t = \tau_{TOR} + \tau_{DIR}$$

$$\Rightarrow \tau_t = \frac{P}{A}\left(1 + \frac{2R}{r}\right)$$

$$\Rightarrow \boxed{\tau_t = \frac{P}{A}\left(1 + \frac{2R}{r}\right)}$$

r = [redacted] wire radius

R = spring radius

$C = \frac{R}{r}$ = spring Index

→ Deflection of spring:

$$\boxed{\delta = \frac{8PD^3 N_a}{G d^4}}$$

Here, N_a = No of active turn.

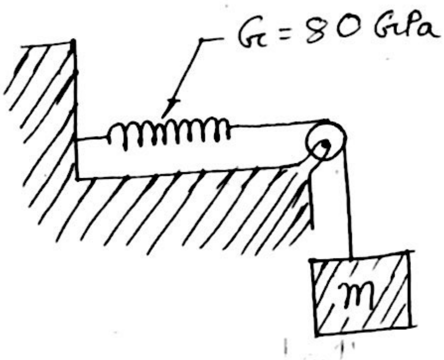
$$\boxed{N_a = N_t - N_i}$$

N_t: Total turn; N_i: Inactive turn.

→ No of inactive turn depends on end condition.

End condition	N_i
Free end	0
Squared (closed)	2
Ground (Machined)	1
Ground and squared	2

Problem: (32) A spring is being used to hold a mass 'm' as shown in the figure. If the spring has total turn of 10 and diameter of 50 mm, then find out the maximum mass that the spring can hold. The shear strength of the spring material is 200 MPa. The spring is made of 10 mm wire.



Solution: Let, maximum mass that the spring can support is m kg.

$\therefore$ maximum shear stress developed.

$$\tau = \frac{P}{A}\left(1 + \frac{2R}{r}\right) = \frac{m \times 9.81}{\pi \times 5^2}\left(1 + \frac{2 \times 25}{5}\right)$$

$$= 1.374(m) \text{ MPa.}$$

Now, $1.374(m) = 200$

$\Rightarrow m = 145.6$ kg. (Ans)

---

* Deflection (Elongation) of spring at $m = 145.6$ kg.

$$\delta = \left(\frac{8 \times 145.6 \times 9.81 \times 50^3 \times 10}{80 \times 10^3 \times 10^4}\right) = 17.85 \text{ mm.}$$

Springs in combination:

- Two or more spring can work to gether in variety of configurations.
- Two spring in parallel divide force into them so that their deformation remain same.
- Two spring in series work under same load / Force.

Problem: (33) Two con-centric springs has diameter or 100 mm and 75 mm. The wire diameter is 12 mm for both the springs. If they are subjected to a load of 450 N then find maximum stress developed in each spring. $\boxed{N_a^o = 1.5 N_a^i}$

Solution: They divide force among themselves.

$$P_o + P_i = 450 \; ; \quad \delta_o = \delta_i$$

we know, $\delta = \dfrac{8FD^3N_a}{Gd^4}$; $\delta \propto (FD^3N_a)$.

$$\therefore \frac{\delta_o}{\delta_i} = \frac{P_o D_o^3 N_a^o}{P_i D_i^3 N_a^i} = 1$$

$\Rightarrow P_i = 3.56 P_o$; $\boxed{P_o + P_i = 450}$

$\therefore P_i = 351.3$ (N) ; $P_o = 98.7$ N

So, stresses:

$$\tau_o = \frac{98.7 \times 4}{\pi (12)^2}\left(1 + \frac{2 \times 100}{12}\right) = 15.41 \text{ MPa}$$

$$\tau_i = \frac{351.3 \times 4}{\pi \times (12)^2}\left(1 + \frac{2 \times 75}{12}\right) = 41.93 \text{ MPa.}$$

Combined Stress:

This type of stress is combination of ~~either~~ ~~or~~ normal & shear stresses in different or same direction.

→ Four possible combination of different loading are:

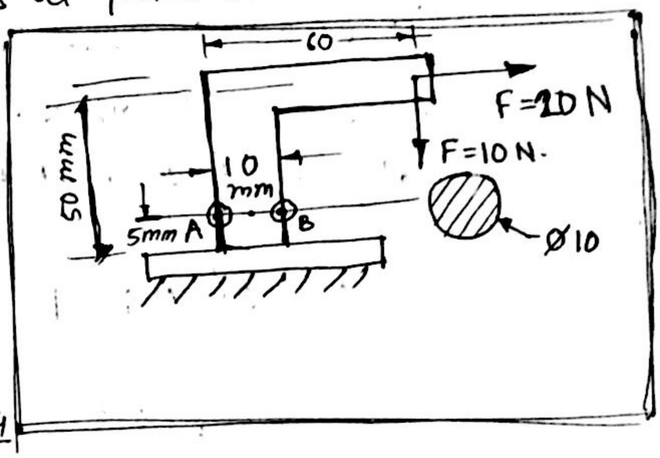
ⓐ Tension / compression + Bending

ⓑ Tension / compression + Torsion

ⓒ Torsion + Bending

ⓓ Tension / compression + Bending + Torsion.

Problem: (34). Find the total stress at point A and B.

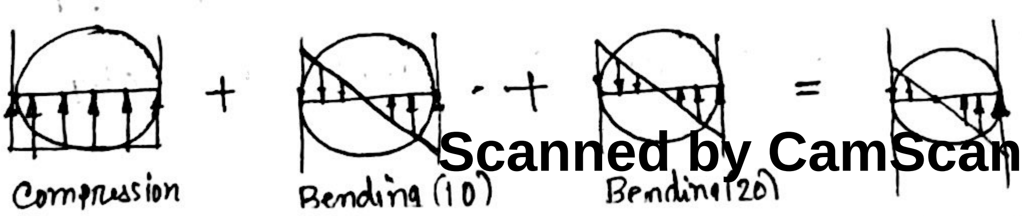


Point A: In compression due to 10 N and in tension due to bending for 20 N and 10 N force.

$$\sigma_A = -\left\{\frac{10\times4}{\pi(10)^2}\right\} + \frac{20\times45\times5\times64}{\pi\times(10^4)} + \frac{10\times60\times5\times64}{\pi\times(10^4)} = -0.127+9.167+6.111 = 15.15 \text{ MPa}$$

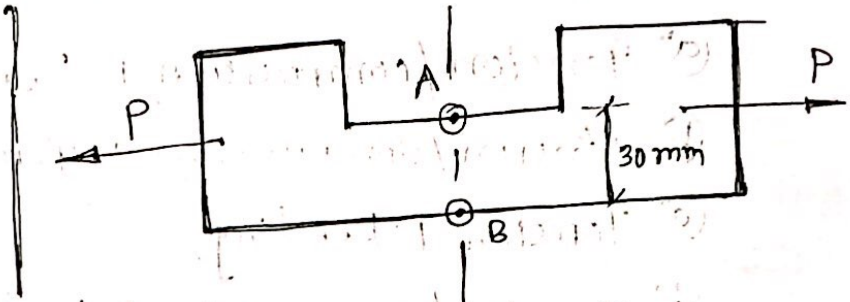
$$\sigma_B = -\left[+\left\{\frac{10\times4}{\pi\times10^2}\right\} + \frac{20\times45\times5\times64}{\pi(10^4)} + \frac{10\times60\times5\times64}{\pi\times(10^4)}\right] = -15.41 \text{ MPa}$$

* Combination of bending and compression/tension.



Problem: (35) Determine the largest force that the link (shown in figure) can support. The thickness of the link is 50 mm.

The strength of the material is 500 MPa.



Solution:

Point A will be subjected to tension and tensile flexural stress. While point B will be subjected to tensile and compressive flexural stress. So stress at point A will be maximum. $A = 30 \times 50 = 1500 \text{ mm}^2$

Stress at A: $\sigma = \frac{P}{A} + \frac{Mc}{I}$

$$\Rightarrow 500 = \frac{P}{1500} + \frac{(P \times 15) \times 15 \times 12}{50 \times (30)^3}$$

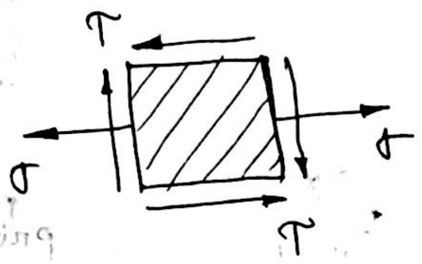
$$\Rightarrow 500 = \frac{4P}{1500}$$

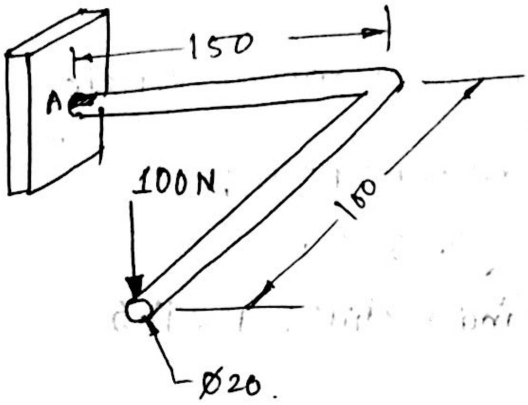
$\therefore P = 187500 \text{ N} = 187.5 \text{ KN}$

* Same type of stress in same plane.
* Some time different type stress on a single element.

Combination of different stress: Consider the link in figure. Imagine an elementary section A.

Element A:





point 'A' is subjected to two different type stress in two different direction.

- Remember, stress is a vector quantity.
- Never add or substract shear and normal stress in a conventional arithmatic manner.
- For this case we use mohr's circle.

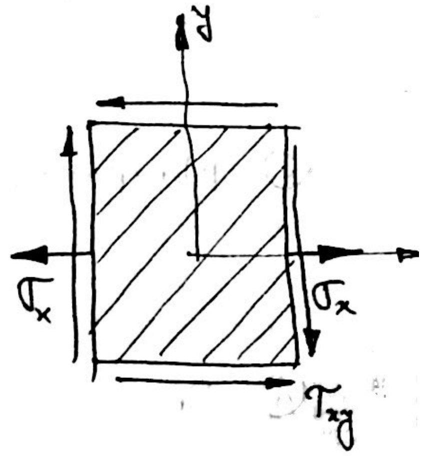
2D Mohr's circle:

- x and y axes represent normal and shear stresses
- Angle in circle is twice the actual ~~angle~~ angle.

Consider element A:

$$\sigma_x = \frac{Mc}{I} = \frac{100 \times 150 \times 10 \times 64}{\pi (20)^4} = 19.1 \text{ MPa}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{100 \times 160 \times 10 \times 32}{\pi (20)^4} = 6.4 \text{ MPa}$$



→ Draw a mohr's circle.

→ Find the center:

(9.55, 0)

→ Find radius: R = 11.5

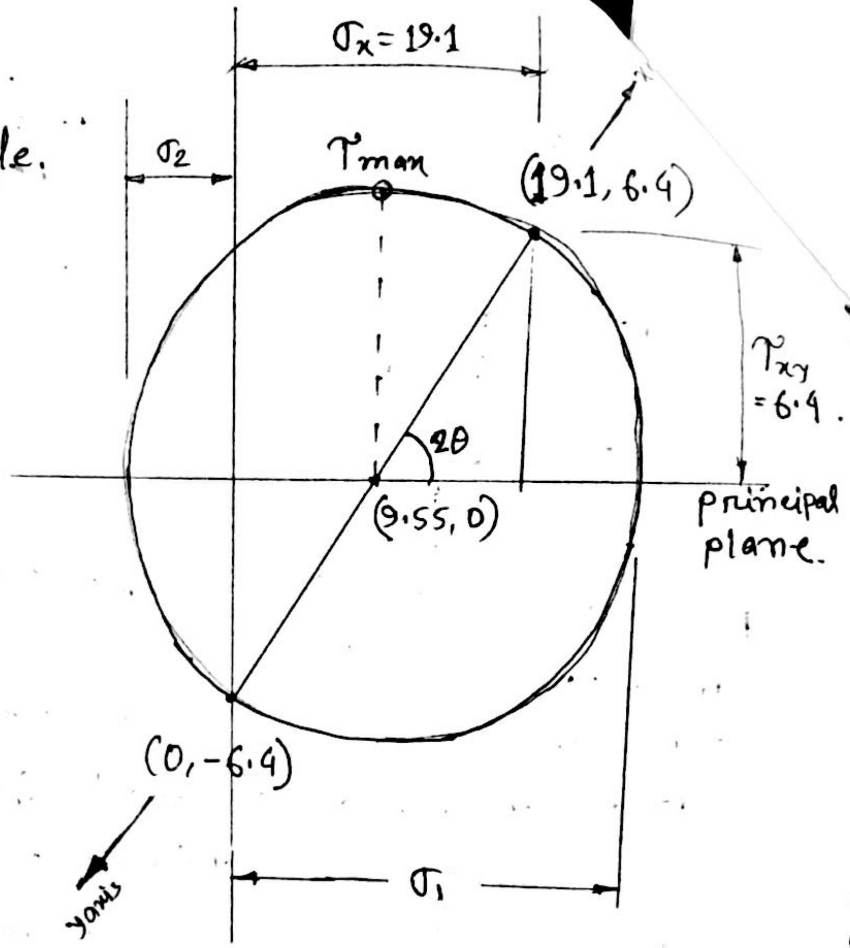
To find:

ⓐ Principal plane

ⓑ Principal stress

ⓒ Max. Shear stress

ⓓ Stress along $\theta = 45^\circ$



ⓐ Principal plane makes an angle 2θ with x axis in mohr's circle, $\therefore \tan 2\theta = \frac{6.4}{19.1 - 9.55} \Rightarrow \theta = -16.9^\circ$

ⓑ Principal stress: (σ_1)

$$\sigma_1 = 9.55 + R = 9.55 + 11.5 = 21.05 \text{ MPa.}$$

$$\sigma_2 = 9.55 - R = 9.55 - 11.5 = -1.95 \text{ MPa.}$$

ⓒ Maximum shear stress: (τ_{max})

$$\tau_{max} = +R = 11.5 \text{ MPa.}$$

ⓓ Draw a line that make $(45 \times 2) = 90^\circ$ with the x axis in mohr's circle.

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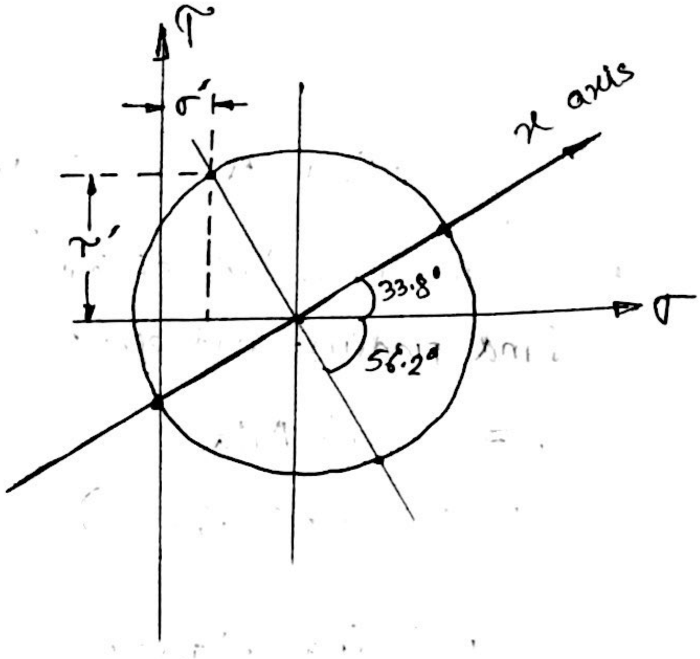
Now, $\sigma' = 9.55 - R\cos(56.2)$

$= 9.55 - 11.5\cos(56.2)$

$= 3.15$ MPa.



$\tau' = R\sin(56.2)$

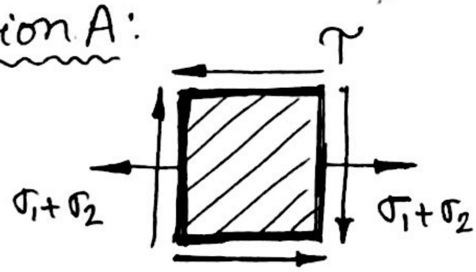
$= 11.5\sin(56.2)$

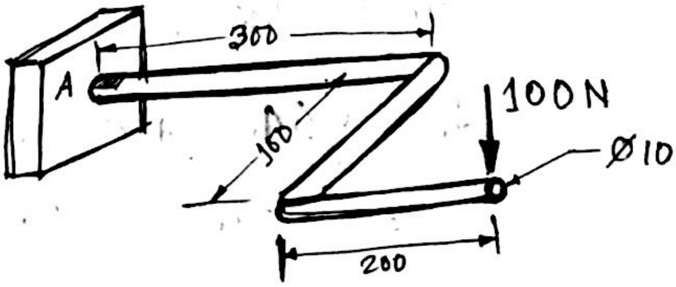
$= 9.56$ MPa.

Problem: (36) For the element shown in figure find the principal stress and principal plane also find the value of maximum shear stress.

Solution:

Section A:



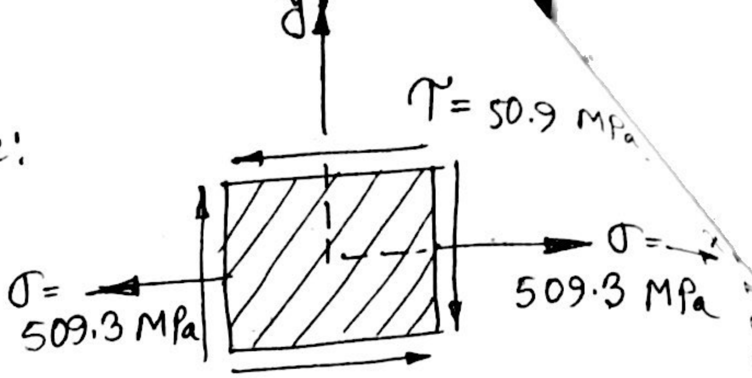


Here, $\sigma_1 = \left(\dfrac{Mc}{I}\right) = \dfrac{(100\times 300)\times 5\times 64}{\pi(10)^4} = 305.6$ MPa.

$\sigma_2 = \left(\dfrac{Mc}{I}\right) = \dfrac{(100\times 200)\times 5\times 64}{\pi\times(10)^4} = 203.7$ MPa.

$\tau = \left(\dfrac{Tc}{J}\right) = \dfrac{(100\times 100)\times 5\times 32}{\pi\times(10)^4} = 50.9$ MPa.

So, section A. looks like:

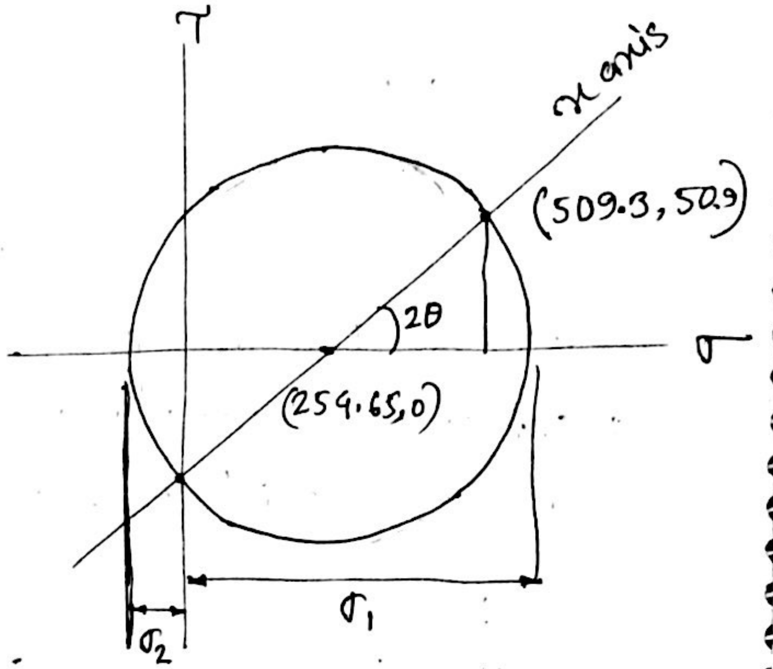


→ Draw a mohr's circle

→ Find radius and center.

$R = 259.7$ MPa.

Center $= (254.65, 0)$



Now, principal plane;

$$\sin 2\theta = \left(\frac{50.9}{259.7}\right)$$

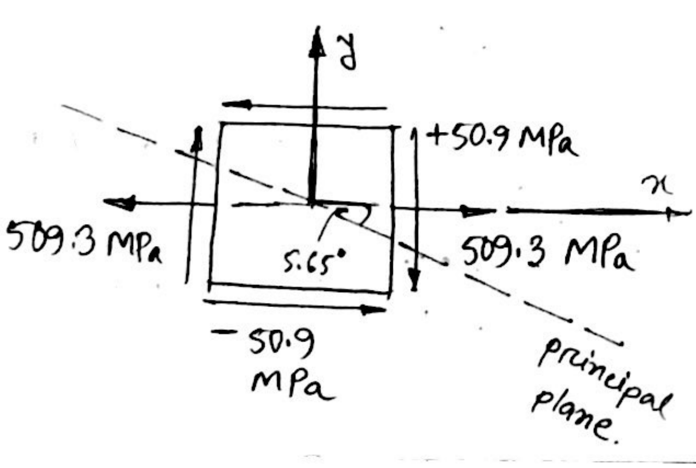
$\Rightarrow \theta = 5.65°$ (negative)

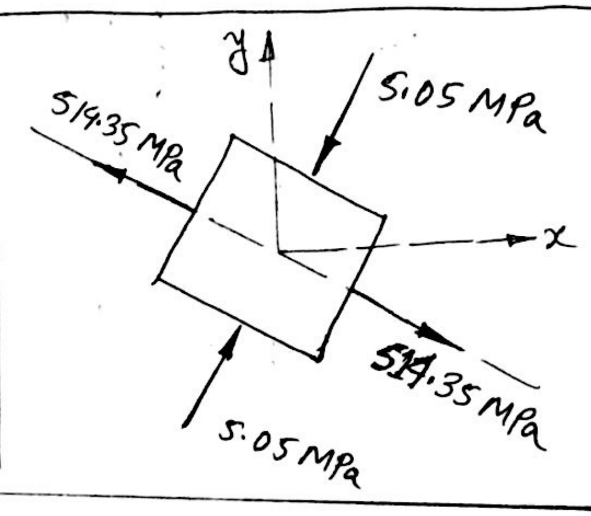
principal stress:

$$\sigma_1 = (254.65 + 259.7)$$

$$= 514.35 \text{ MPa}$$

$$\sigma_2 = (254.65 - 259.7) = -5.05 \text{ MPa}.$$

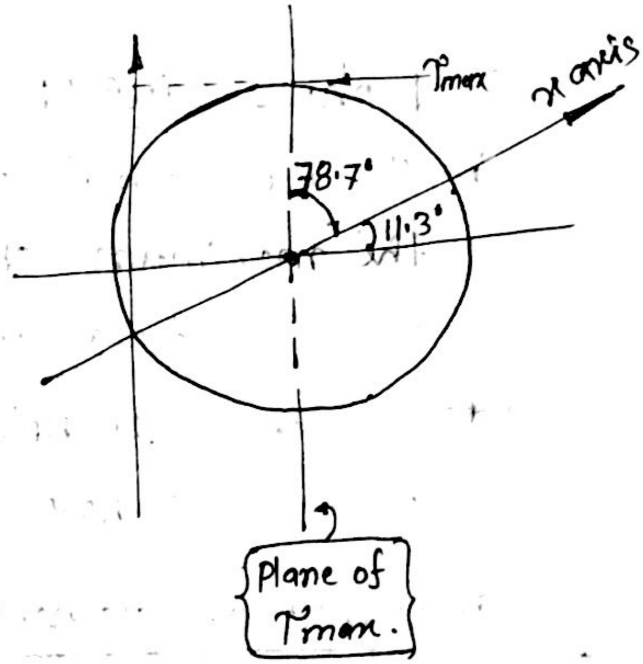




Plane of maximum shear:

$2\theta = (90 - 11.3) = 78.7°$

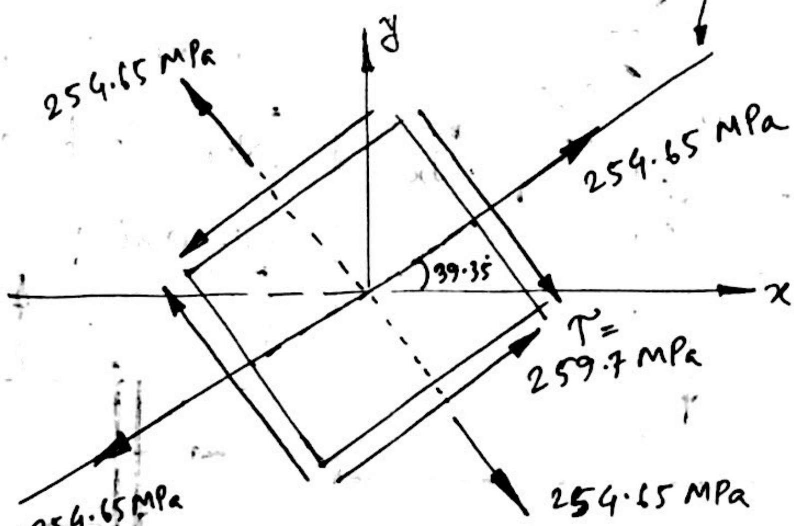
$\Rightarrow \theta = 39.35°$



Stresses on plane of maximum shear:

$\tau_{max} = 259.7$ MPa

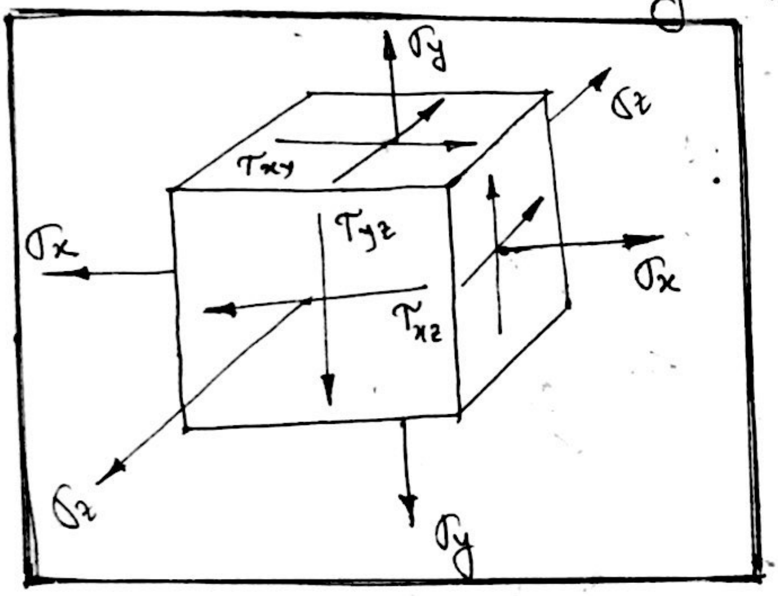
$\sigma' = 254.65$ MPa.



3D Mohr's circle:

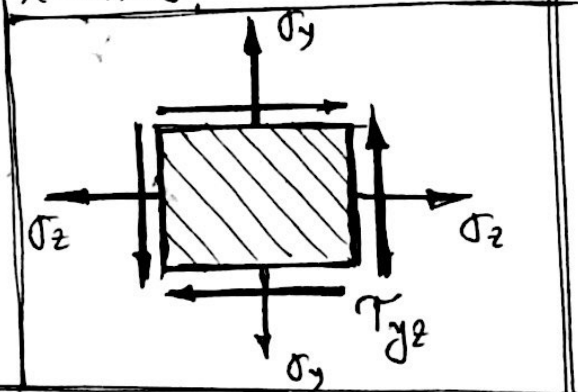
- The prime objective of drawing mohr's circle is to find the maximum shear stress developed. As almost every material are weaker in shear.
- So, in 3D mohr's circle we will just study the maximum shear stress.

Consider an elementary volume shown in figure:

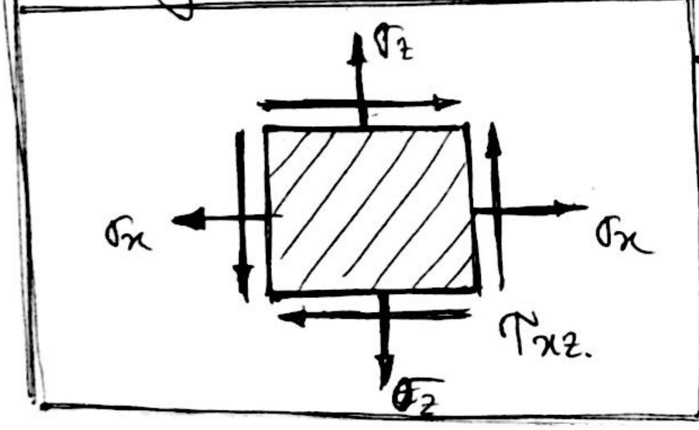


All possible stresses are shown in the figure.

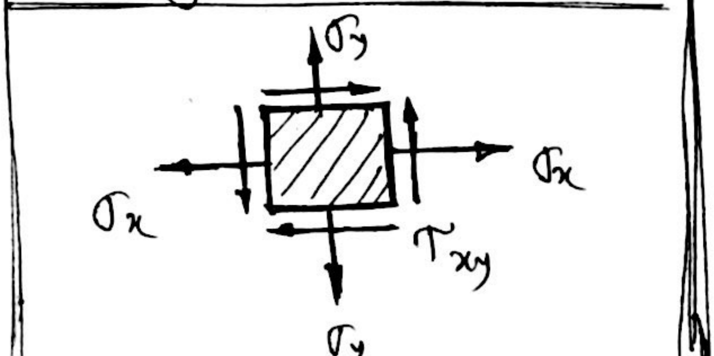
Now looking from x-axis:



looking from y axis:

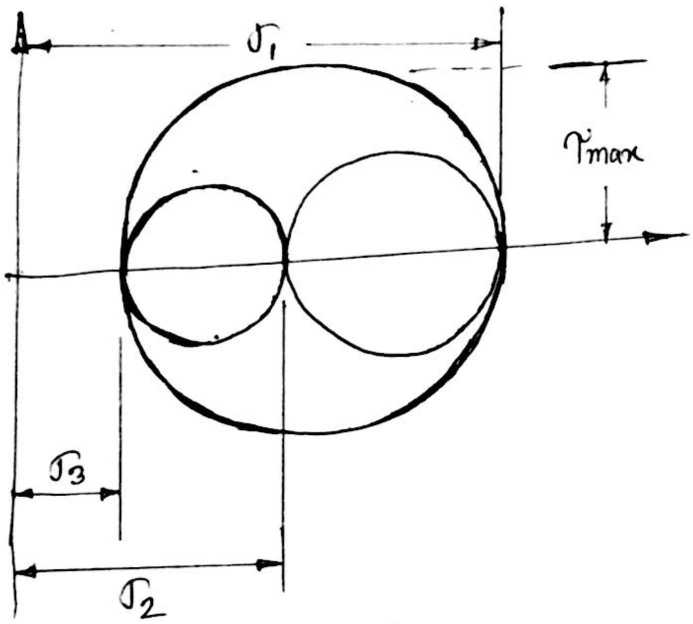


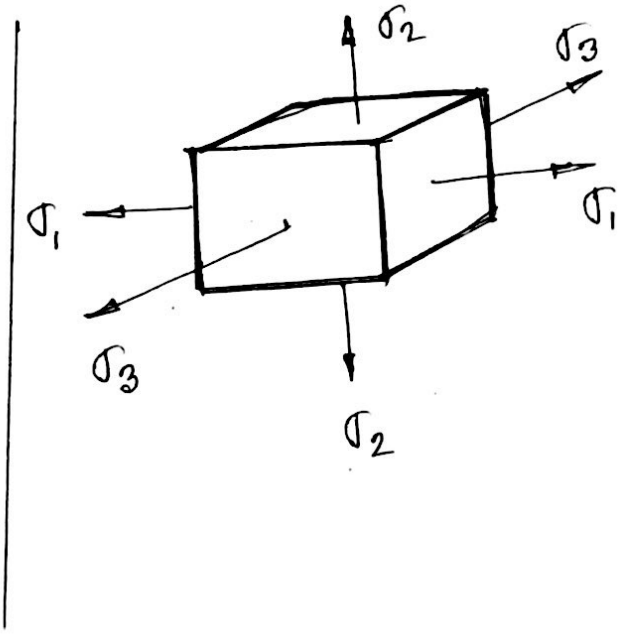
Looking from z axis



→ From above three figure we can find 3 different principal stress, σ_1 σ_2 and σ_3.

→ Now we can draw a 3-D mohr's circle. with three possible combination.

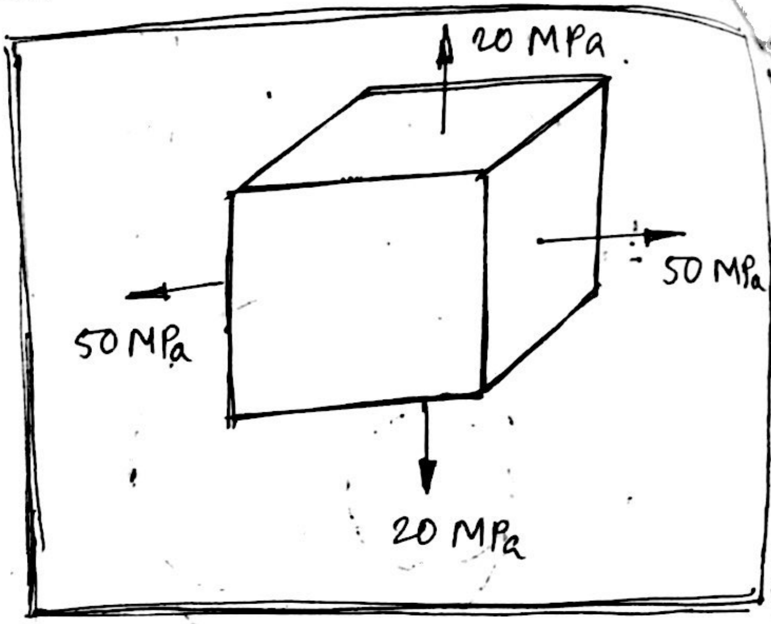




→ So, $\tau_{max} = \left|\frac{\sigma_1 - \sigma_2}{2}\right|$ or $\left|\frac{\sigma_1 - \sigma_3}{2}\right|$ or $\left|\frac{\sigma_2 - \sigma_3}{2}\right|$; the larger one is the maximum shear stress.

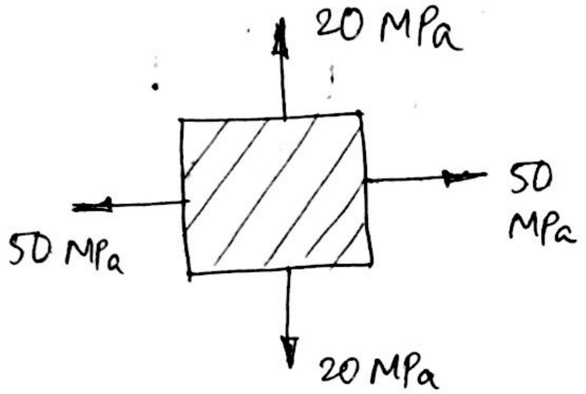
→ After having τ_{max} we can proceed to design.

Problem: (37) Find the in plane ~~stre~~ maximum shear stress and absolute maximum shear stress.



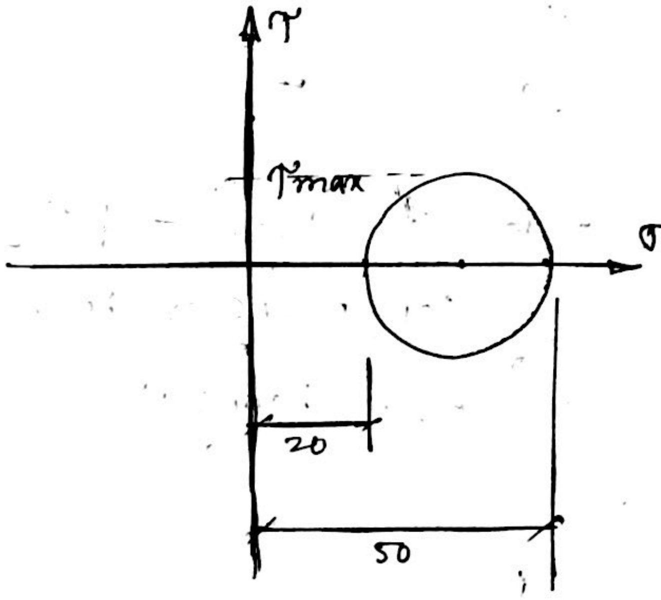
Solution:

ⓐ In plane maximum Shear stress:

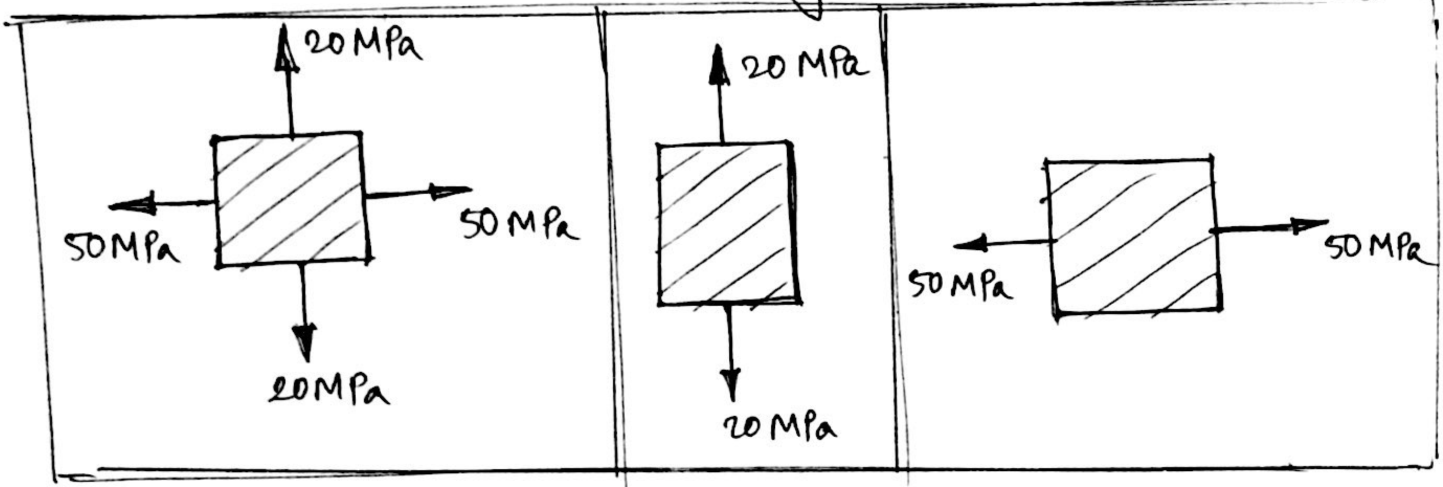


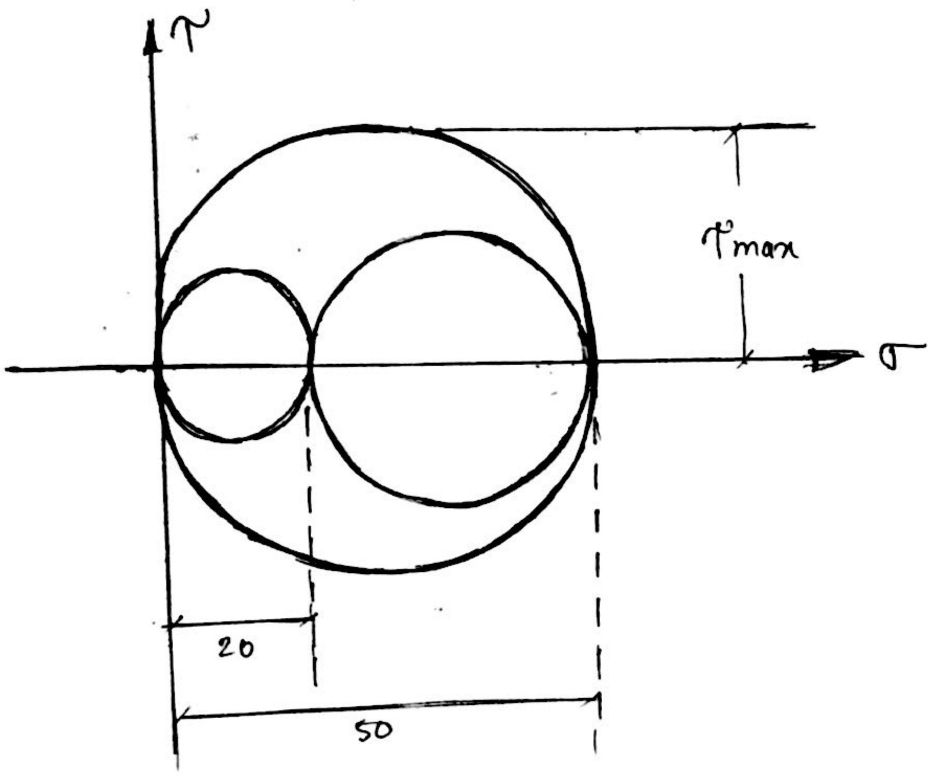
$$\tau_{max} = \pm\left[\frac{50-20}{2}\right]$$

$$= \pm 15 \text{ MPa}.$$



ⓑ Absolute maximum shearing stress.





$\tau_{max} = (50 - 0)/2 = 25$ MPa.

Column:

- Always under axial compressive load.
- Up to a certain level act like a compression block.
- When length become sufficiently large fails by buckling and are called column. $\boxed{l/d \geq 10}$
- Columns are two types:
  - (a) long column → fails by buckling.
  - (b) Intermidiate column → fails by buckling and compression.
- For long column Euler's formula are applicable.
- For Intermidiate column Jhonsons formula is applicable
- Wheathar a column is Euler column or Jhonsons column is determined by slenderness ratio. (l/k).
- For euler column: $\boxed{(l/k)_{actual} \geq (l/k)_{critical}}$

  $(l/k)_{actual}$ → depends on ~~material and~~ dimension's of the ~~beam~~. column.

  $(l/k)_{critical}$ → depends on material and dimension of the column.

- Euler equation: $\boxed{\frac{P}{A} = \frac{\pi^2 CE}{(l/k)^2}}$

- Johnsons' equation: $\boxed{\frac{P}{A} = \sigma_y - \left(\frac{\sigma_y}{2\pi}\right)^2 \frac{1}{CE}\left(\frac{l}{k}\right)^2}$

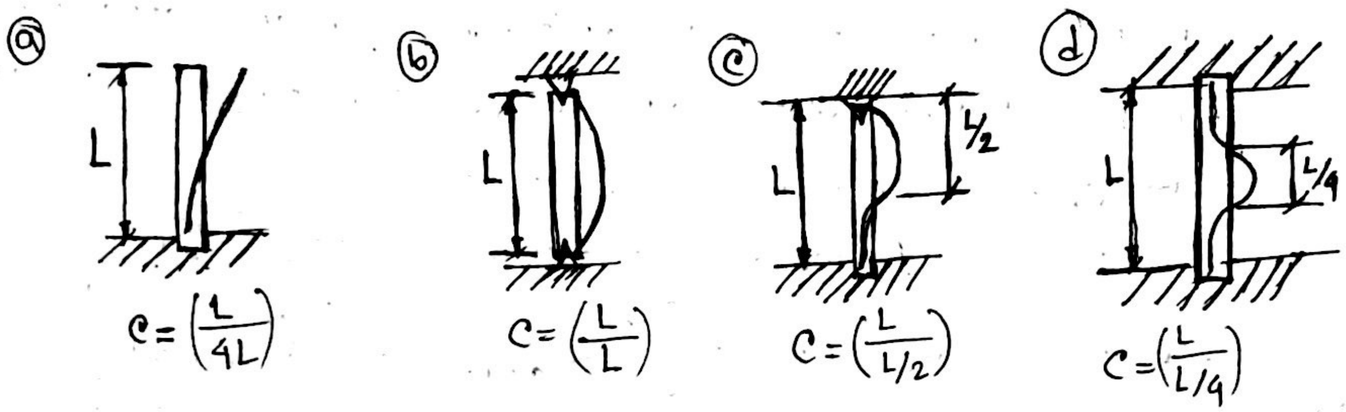
- (l/k) critical : $\boxed{(l/k)_{cr} = \sqrt{\left(\frac{2\pi^2 CE}{\sigma_y}\right)}}$

→ Here, C → end condition constant. depends only on end of the column

→ Basically C is the ratio of column length to the length that actually act like a column.

End condition	C
Fixed - free	1/4
Hinged - Hinged	1
Fixed - Hinged	2
Fixed - Fixed	4

So, $\boxed{C = \frac{L}{L_e}}$ | L = Length of column
L_e = Effective length as column

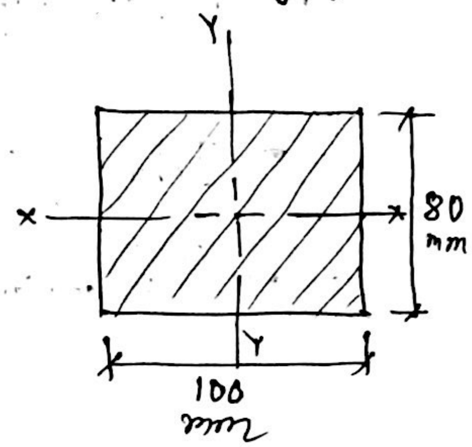


Problem: (38) A rectangular column of length 2■m is to be subjected to a pure axial load. If the strength of the material is 400 MPa and E = 200 GPa find the maximum load that it can support. (both end hinged)

Solution: $\left(\frac{l}{k}\right)_{cr} = \sqrt{\frac{2\pi^2 CE}{\sigma_y}}$

$= \sqrt{\frac{2\pi^2 \times 1 \times 200 \times 10^3}{400}}$

$= 99.3$



→ this is the limit for $(l/k)_{actual}$. If $(l/k)_{actual} \geq 99.3$ then its long column. else its intermediate column.

$$(l/k)_{ac} = \left(\frac{2000}{\sqrt{\frac{4266666}{8000}}} \right)$$

$$= 69.3.$$

So, this is an intermidiate column.

$\boxed{I = I_{min}}$

$$I_{xx} = \frac{1}{12} \times (100 \times 80^3)$$
$$= 4266666 \ mm^4.$$
$$I_{YY} = \frac{1}{12}(80 \times 100^3)$$
$$= 6666666 \ mm^4.$$
$$\therefore I = I_{xx} = 4266666 \ mm^4$$

$$\therefore \frac{P}{A} = \sigma_y - \left(\frac{\sigma_y}{2\pi}\right)^2 \frac{1}{CE} \left(\frac{l}{k}\right)^2$$

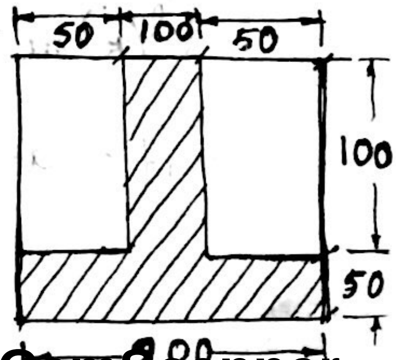
$$\Rightarrow P = 80000 \left[400 - \left(\frac{400}{2\pi}\right)^2 \frac{1}{1 \times 200 \times 10^3} (69.3)^2 \right] = 2421500 \ N$$
$$= 2421.5 \ kN.$$

Problem: (39) A column is one end hinged and one end fixed and has a length of 20 m. $\sigma_y = 450$ MPa; $E = 200$ GPa. Find the maximum load it can support.

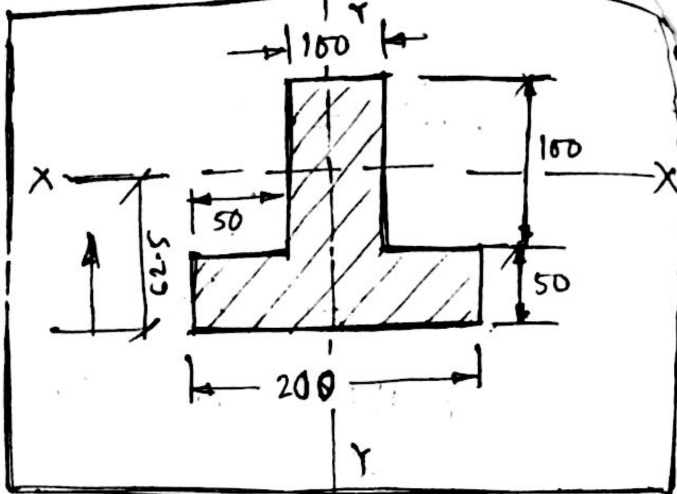
Solution: $(l/k)_{cr} = \sqrt{\frac{2\pi^2 C E}{\sigma_y}}$

$$= \sqrt{\frac{2 \times \pi^2 \times 2 \times 200 \times 10^3}{450}}$$

$$= 132.5$$



Now, if actual $(l/k)_{ac}$ is greater than 132.5 then it an Euler column. Else it is a Johnson's column



$$\bar{y} = \frac{\Sigma A \bar{y}}{\Sigma A}$$

$$= \frac{(50\times200)\times25 + (100\times100)\times100}{(50\times200 + 100\times100)}$$

$$= 62.5$$

$$I_{xx} = \left[\frac{1}{12}(200\times50^3) + 50\times200\times(37.5)^2\right] + \left[\frac{1}{12}\times(100\times100^3) + (100\times100)\times(37.5)^2\right]$$

$$= 57291666 \text{ mm}^4$$

$$I_{YY} = \left[\left(\frac{1}{12}\,100\times100^3\right) + \left(\frac{1}{12}\times50\times200^3\right)\right] = 41666666 \text{ mm}^4$$

$$I = I_{YY} = 41666666 \text{ mm}^4$$

So, $K = \sqrt{\frac{I}{A}} = \sqrt{\frac{41666666}{2000}} = 144$ mm.

$$\therefore (l/k)_{ac} = \left(\frac{20\times10^3}{144}\right) = 138.9.$$

So it's an Euler column. $(l/k)_{ac} > (l/k)_{cr}$.

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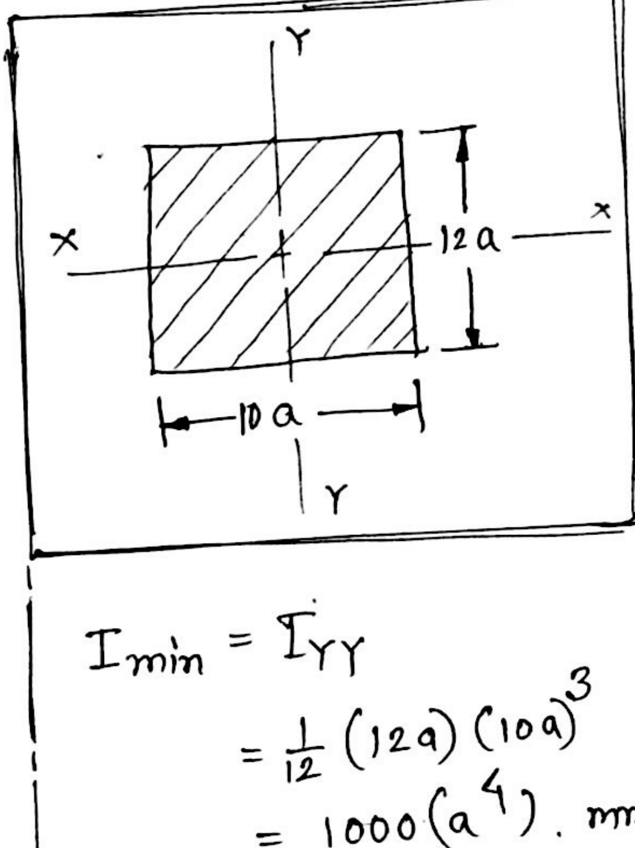
So, $\dfrac{P}{A} = \dfrac{\pi^2 C E}{(l/k)^2}$

$$\Rightarrow P = 2000\left[\frac{\pi^2 \times 2 \times 200\times10^3}{\cancel{}\,(138.9)^2}\right] = 409248\ N$$

$$= 409.2\ kN.$$

<u>Problem:(40)</u> A column is to be designed to support a load of 5000 kN. the length of the column is 20 m. material strength and 'E' are 400 MPa and 200 GPa. Find the dimensions. $\boxed{\text{Both end hinged}}$



<u>Solution:</u>

$$(l/k)_{cr} = \sqrt{\frac{2\pi^2 C E}{\sigma_y}}$$

$$= \sqrt{\frac{2\pi^2 \times 1 \times 200\times10^3}{400}}$$

$$= 99.3.$$

$*$ $(l/k)_{actual} \rightarrow \boxed{\text{(Unknown) depends on 'a'}}$

Assume the column as long:

$$\therefore \frac{P}{A} = \frac{\pi^2 C E}{(l/k)_{ac}}$$

$$\Rightarrow (l/k)_{ac} = \frac{\pi^2 \times 1 \times 200\times10^3 \times 120a^2}{5000\times10^3}$$

$$\Rightarrow \frac{2400}{a^2} = \frac{\pi^2 \times 24\times10^6 . a^2}{5\times10^6}$$

$$I_{min} = I_{YY}$$

$$= \frac{1}{12}(12a)(10a)^3$$

$$= 1000(a^4).\ mm^4$$

$$k = \frac{1000a^4}{120a^2} = 8.33\,a^2$$

$$(l/k)_{ac} = \left(\frac{20\times10^3}{8.33\,a^2}\right) = \frac{2400}{a^2}.$$

$\Rightarrow$ $a = 2.66$ mm.

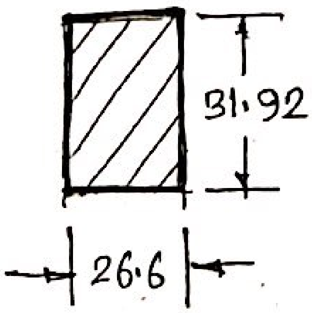
Now, $\left(l/k\right)_{ae} = \dfrac{2400}{(2.66)^2} = 339.1 > 99.3.$

$\boxed{\text{So, our assumption is correct.}}$

$\therefore$ Dimension of the column:

$$\boxed{\begin{aligned} a &= 2.66 \text{ mm} \\ 10a &= 26.6 \text{ mm} \\ 12a &= 31.92 \text{ mm} \end{aligned}}$$

(Ans).

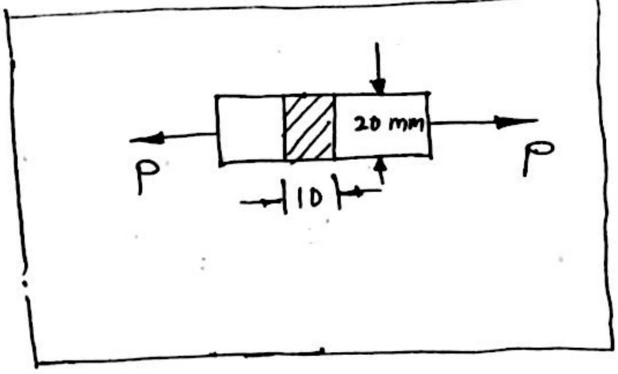


Factor of safety: (Safety factor) (N)

→ This is the ratio of maximum load a member can support (Depends on strength) to applied load.

→ If a member can support 2 kN (with no safety factor) then it can support 1 kN with a factor of safety 2.

Problem: (41) Determine the maximum load 'p' that the member can support ⓐ without any factor of safety (N=1) ⓑ with a factor of safety N = 3. $\boxed{\sigma_y = 200 \text{ MPa}}$



Solution:

ⓐ Developed stress:

$$\sigma = \left(\frac{P}{200}\right) \text{ MPa}$$

Now, for factor of safety (N=1)

$$\sigma = \sigma_y \; ; \; P = 40 \text{ KN} \text{ (Ans)}$$

ⓑ Developed stress:

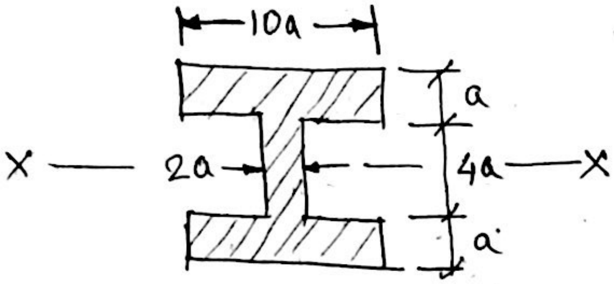
$$\sigma = \left(\frac{P}{200}\right) \text{MPa}$$

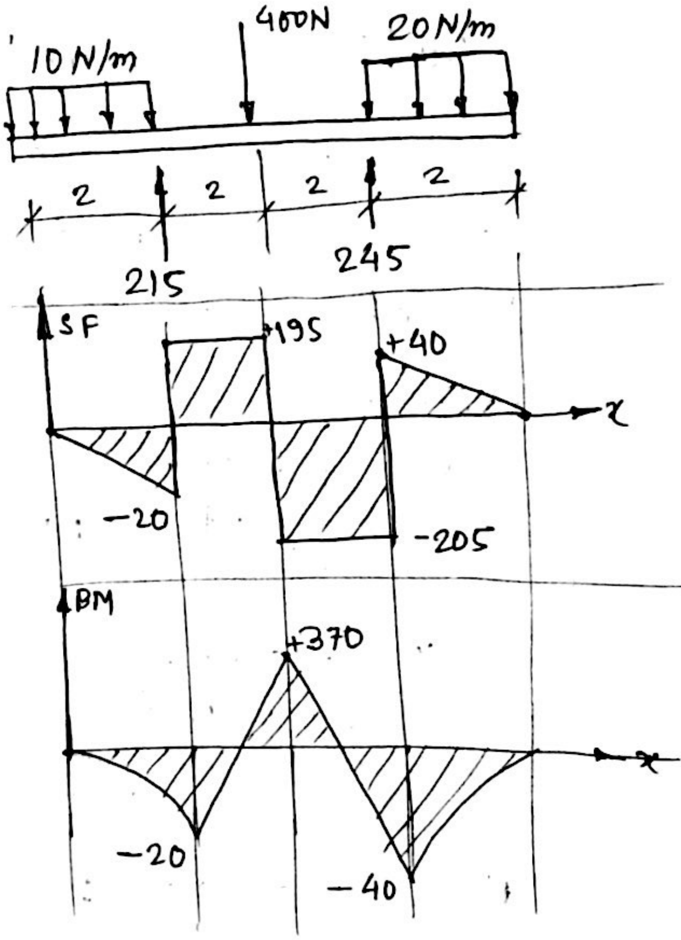
Now, for factor of safety (N=2)

$$\sigma = \frac{\sigma_y}{2} = \frac{P}{200}$$

$$\Rightarrow \quad P = 20 \text{ KN. (Ans)}$$

Problem: (42) Find the dimensions of the beam if the flexural strength of beam material is 200 MPa. Safety factor $N = 3$. (Repeat of problem: 16)





$I_{xx} = 137.3\,a^4 \text{ mm}^4$

Maximum developed stress:

$$\sigma = \left(\frac{Mc}{I}\right) = \frac{370 \times 10^3 \times 3a}{137.3\,a^4}$$

$$= \left(\frac{8084.5}{a^3}\right)$$

Now, For factor of safety ($N = 3$)

$$N = \frac{\sigma_y}{\sigma}$$

$$\Rightarrow \sigma = \frac{\sigma_y}{3}$$

$$\Rightarrow \frac{8084.5}{a^3} = \frac{200}{3}$$

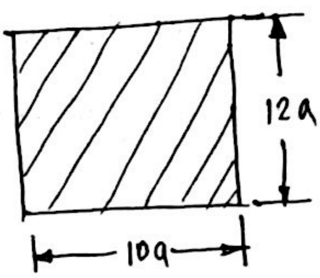
$$\Rightarrow a = 4.95 \text{ mm.}$$

So, incorporating $N > 1$ makes the structure more safe.

Problem:(43) A column is to be designed to support a load of 5000 kN. The length of the column is 20 m. Material strength and E are 400 MPa and 200 GPa Both end hinged. find the dimensions. (Use, N = 3)

Solution:

$$(l/k)_{cr} = \sqrt{\frac{2\pi^2 CE}{\sigma_y}} = 99.3$$



$$I = I_{min} = 1000\ a^4\ mm^4$$

$$K = \sqrt{\frac{I}{A}} = 8.33\ a^2\ mm.$$

$$(l/k)_{ac} = \left(\frac{2400}{a^2}\right).\quad \boxed{\text{Unknown}}$$

Assume Euler column: $\frac{P}{A} = \frac{\pi^2 CE}{(l/k)_{ac}}$

$$\Rightarrow \frac{5000\times10^3\times3}{120a^2} = \frac{\pi^2(1)\times200\times10^3\times a^2}{2400}$$

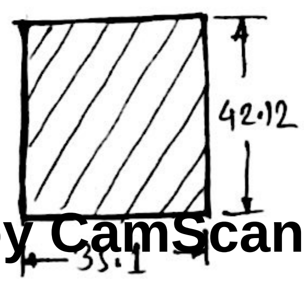
$$\Rightarrow a = 3.51\ mm.$$

$$\therefore (l/k)_{ac} = \left(\frac{2400}{a^2}\right) = \frac{2400}{(3.51)^2} = 194.8. > (l/k)_{cr}.$$

$\boxed{\text{Assumption is correct}}$

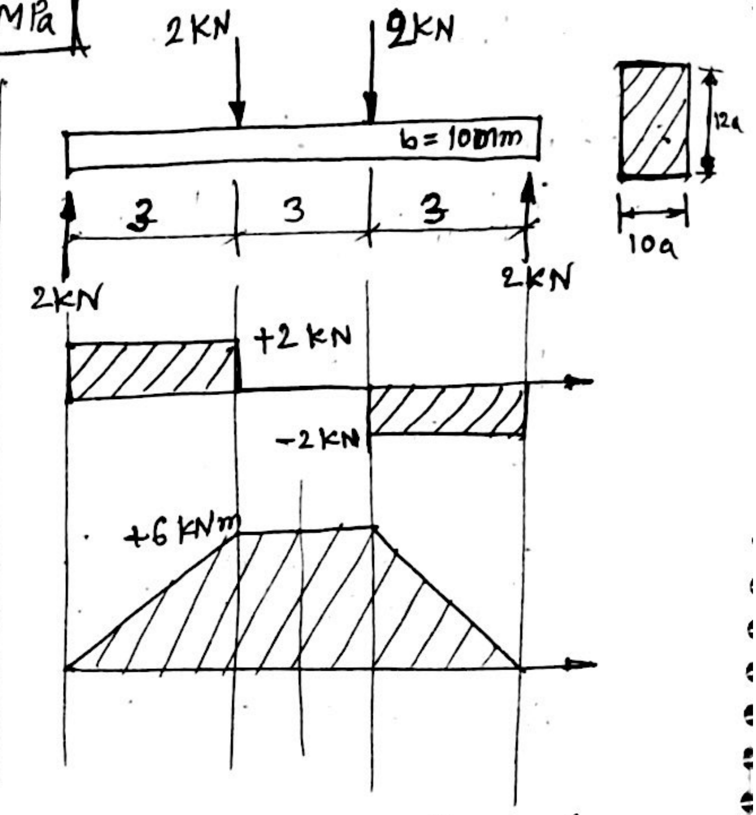
∴ Dimensions are:

$$\boxed{\begin{aligned} a &= 3.51\ mm \\ 10a &= 35.1\ mm \\ 12a &= 42.12\ mm \end{aligned}}$$



Ans.

Problem: (44) Design the beam as shown in the figure. Assume factor of safety $N = 2.5$. (complete design) the deflection ~~at free end~~ must not exceed 0.005 mm. at any point. $\boxed{\sigma_y = 400 \text{ MPa}, \tau_y = 180 \text{ MPa}}$



Solution:

(a) Design based on stress:

Flexural stress:

$$\therefore \sigma = \frac{Me}{I}$$

$$= \frac{6\times10^6 \times 6a}{1440\, a^4}$$

$$= \left(\frac{25000}{a^3}\right)$$

This stress is on upper or lower fiber of the beam. There is no other stress.

So, $N = \frac{\sigma_y}{\sigma} \Rightarrow 2.5 \times \left(\frac{25000}{a^3}\right) = 400$; $a = 5.38$ mm

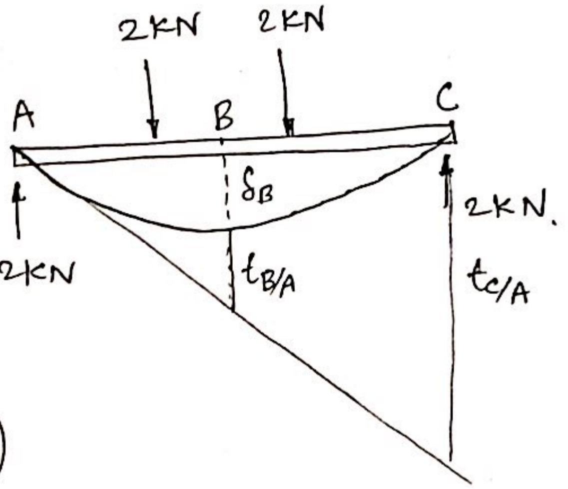
Shear stress:

$$\tau = \frac{VQ}{Ib} = \frac{2\times10^3 \times 180\, a^3}{1440\, a^4 \times 10} = \frac{25}{a} \text{ MPa}.$$

There is no other stress along neutral axis.

$N = \frac{\tau_y}{\tau} \Rightarrow \frac{25}{a} = \frac{180}{2.5}$; $a = 0.34$ mm.

(b) Design based on deflection:



As the beam is symmetric loaded. the maximum deflection is at mid span where ($x = 4.5$ m).

$$t_{B/A} = \left[\left(\frac{1}{2}\times3\times6\right)\times 2.5\right]\left(\frac{1}{EI}\right) + \left[(6\times1.5)\times 0.75\right]\left(\frac{1}{EI}\right)$$

$$= \frac{1}{EI}(29.25) = \left(\frac{29.25}{200\times10^3\times1440\,a^4}\right) = \left[\frac{1}{9846153.8\,a^4}\right].$$

$$t_{C/A} = \frac{1}{EI}\left[\left(\frac{1}{2}\times3\times6\right)\times7 + (6\times3)\times4.5 + \left(\frac{1}{2}\times3\times6\right)\times2\right]$$

$$= \frac{162}{EI} = \left[\frac{1}{1777777.8\,a^4}\right]$$

$$\therefore \frac{\delta_B + t_{B/A}}{4.5} = \frac{t_{C/A}}{9}$$

$$\Rightarrow \delta_B = \left(\frac{t_{C/A}}{2} - t_{B/A}\right) = \frac{1}{EI}(81 - 29.25) = \frac{51.75}{EI}$$

$$= \left(\frac{1}{5565217.4\,a^4}\right)$$

Now, Deflection must be less than 0.005 mm.

$$\therefore \left\{\frac{1}{556\cancel{3}5217.4(a^4)}\right\} = 0.005 \;;\quad a = 0.07 \text{ mm.}$$

Value of a = 5.38 mm. (A)

Failure theories:

→ There are three theories for a struture failure.

ⓐ Maximum Normal stress theory: (τ_y → Not discussed)

* A member/structure fails when maximum normal stress become higher than strength.

ⓑ Maximum Shear stress theory: ($\tau_y = 0.5\,\sigma_y$)

* A member/Struture fails when maximum shear stress become higher than shear strength.

ⓒ Distorsion Energy theory: ($\tau_y = 0.577\,\sigma_y$)

* A member/struture fails when total strain energy become to equal of strain energy stored in a member of tension test specimen at yield point.

→ According to this theory:

$$\sigma = \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2}{2}}$$

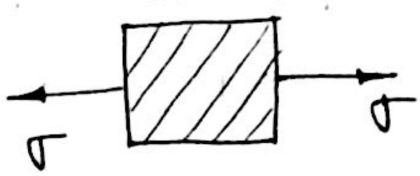
→ For 2D analysis: $\sigma = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2}$

Problem: (45) Find the diameter of the rod shown in figure according to all failure theories. $\sigma_y = 400$ MPa.

Solution: (Assume $N = 2$)

(a) Maximum Normal stress theory:

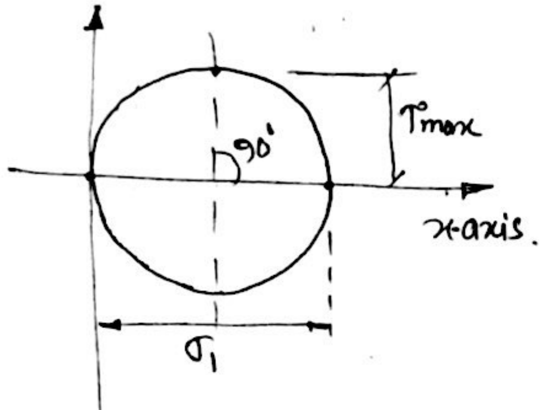
$$\sigma = \frac{F}{A} = \frac{10\times10^3\times4}{\pi(d^2)}$$

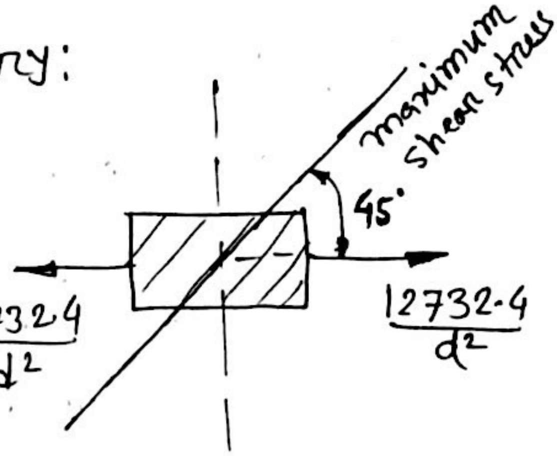


No other stress in the element.

$$\therefore \quad N = \frac{\sigma_y}{\sigma} \Rightarrow \frac{12732.4}{d^2} = \frac{400}{2}; \; d = 8 \text{ mm}.$$

(b) Maximum Shear stress theory:





Though there is no shear applied. But along the 45° plane the shear stress developed is maximum.

$$\tau_{max} = \frac{12732.4}{2d^2} = \frac{6366.2}{d^2}$$

$$\text{Now,} \quad N = \frac{\tau_y}{\tau_{max}} \Rightarrow \frac{6366.2}{d^2} = \frac{(400/2)}{2}$$

$$\Rightarrow d = 8 \text{ mm}.$$

(c) Distorsion Energy theory:

From mohr's circle : $\sigma_1 = \frac{12732.4}{d^2}$ MPa, $\sigma_2 = 0$.

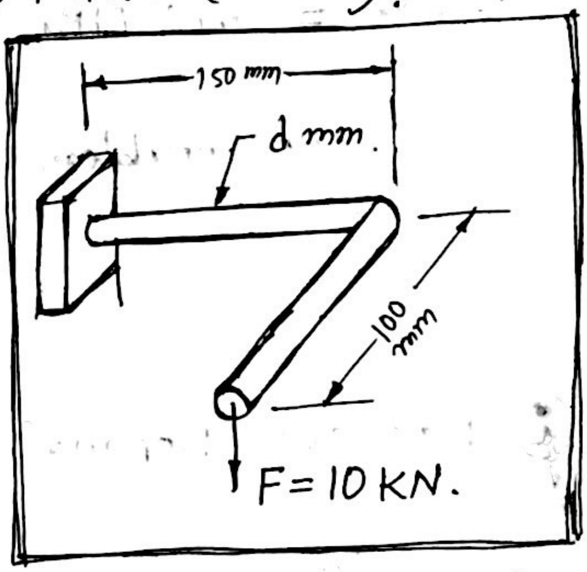
$$\therefore \sigma_e = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2} = \sigma_1 = \frac{12732.4}{d^2}$$

Now, $N = \frac{\sigma_y}{\sigma}$

$$\Rightarrow \frac{12732.4}{d^2} = \frac{400}{2} \; ; \; d \approx 8 \text{ mm}.$$

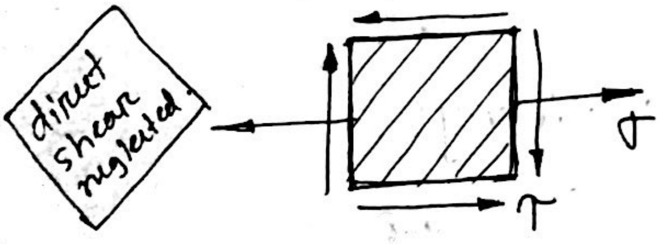
* All theories marges for simple case and yield same results.

Problem : (46) Find the diameter of the shaft if the yield strength = 400 MPa. (N = 3). use all failure theories.



Solution:

(a) Element on upper surface



$$\sigma = \left(\frac{Mc}{I}\right) = \frac{10000 \times 150 \times d/2 \times 64}{\pi(d^4)} = \left\{\frac{15.3 \times 10^6}{d^3}\right\} \text{MPa.}$$

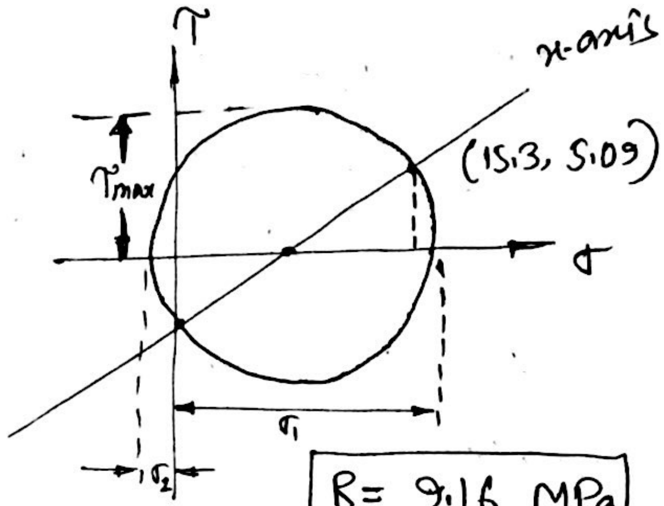
$$\tau = \frac{Tc}{J} = \frac{(10000 \times 100) \times d/2 \times 32}{\pi (d^4)} = \left\{ \frac{5.09 \times 10^6}{d^3} \right\}$$

Draw a mohrs circle with $\sigma = 15.3$ MPa and

$\tau = 5.09$ MPa.

Radius: $R = 9.16$ MPa.

Center: $(7.65, 0)$



$\boxed{R = 9.16 \text{ MPa}}$

$\sigma_1 = 9.16 + 7.65$
$= 16.81$ MPa

$\sigma_2 = 7.65 - 9.16$
$= -1.65$ MPa.

$\tau_{max} = 9.16$ MPa.

So, In our problem, $\sigma_1 = \frac{16.81 \times 10^6}{d^3}$, $\sigma_2 = -\frac{1.65 \times 10^6}{d^3}$

$\tau_{max} = \frac{9.16 \times 10^6}{d^3}$.

(i) Maximum Normal stress theory:

$$N = \frac{\sigma_y}{\sigma_1} \Rightarrow \frac{16.81 \times 10^6}{d^3} = \frac{450}{3}$$

$\Rightarrow d = 50.14$ mm.

(ii) Maximum shear stress theory:

$$N = \frac{\tau_y}{\tau_{max}} \Rightarrow \frac{9.16 \times 10^6}{d^3} = \left(\frac{400/2}{3}\right) T$$

$$\Rightarrow d = 51.6 \text{ mm.}$$

(iii) Distorsion Energy theory:

$$\sigma = \left[\sqrt{(16.81)^2 + (16.81 \times 1.65) + (1.65)^2}\right]\left(\frac{10^6}{d^3}\right)$$

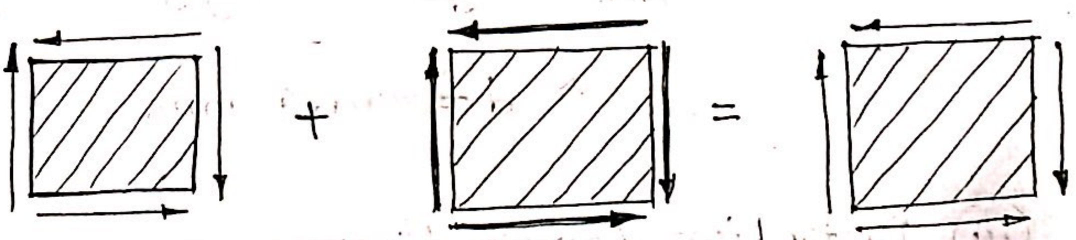
$$= \frac{17.69 \times 10^6}{d^3}$$

$$\therefore \quad N = \frac{\sigma_y}{\sigma} \Rightarrow \frac{17.69 \times 10^6}{d^3} = \frac{400}{3}$$

$$\Rightarrow d = 51.0 \text{ mm.}$$

(b) Element on neutral axis:

$$\boxed{(Q/I) = \left(\frac{\pi d^2}{8} \times \frac{2d}{3\pi}\right) \Big/ \left(\frac{\pi d^4}{64}\right) = \left[\frac{64}{12\pi d}\right]}$$



$$\tau_1 = \frac{VQ}{Ib} = \frac{10 \times 10^3 \times 64}{12\pi \times d \times d} = \frac{16976.5}{d^2}.$$

$$\tau_2 = \frac{Tc}{J} = \frac{5.09 \times 10^6}{d^3}$$

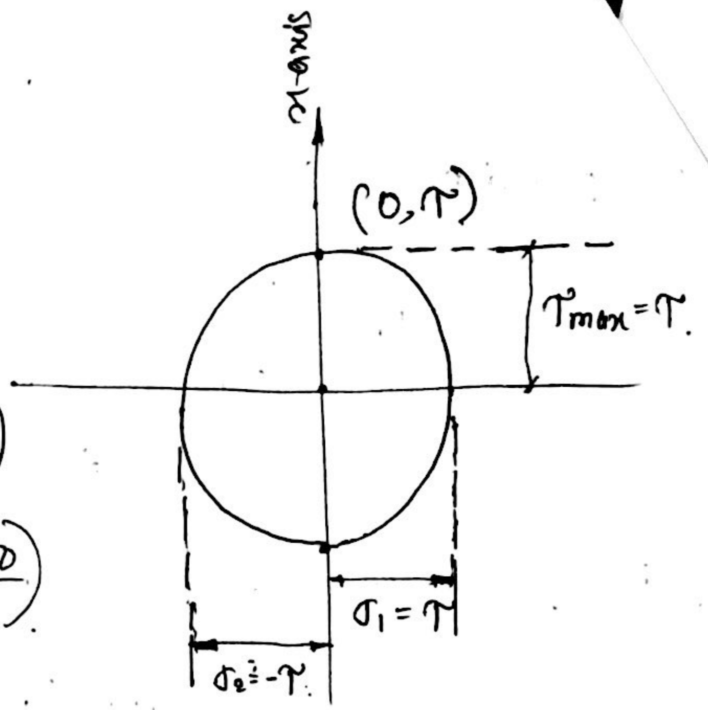
$$\tau = \tau_1 + \tau_2 = \frac{16976.5}{d^2}\left(1 + \frac{300}{d}\right).$$

So, From mohr's circle:

$$\sigma_1 = \tau = \frac{16976.6}{d^2}\left(1 + \frac{300}{d}\right)$$

$$\sigma_2 = -\tau = -\frac{16976.6}{d^2}\left(1 + \frac{300}{d}\right)$$

$$\tau_{max} = \tau = \frac{16976.6}{d^2}\left(1 + \frac{300}{d}\right).$$



(i) Maximum normal stress theory:

$$N = \frac{\sigma_y}{\sigma_1} \Rightarrow \frac{16976.6}{d^2} + \frac{509 \times 10^6}{d^3} = \frac{400}{3}$$

$$\Rightarrow d = 34.9 \text{ mm}.$$

(ii) Maximum shear stress theory:

$$N = \frac{\tau_y}{\tau_{max}} \Rightarrow \frac{16976.6}{d^2} + \frac{5.09 \times 10^6}{d^3} = \frac{200}{3}$$

$$\Rightarrow d = 44.4 \text{ mm}.$$

(iii) Distorsion Energy theory:

$$\sigma = \sqrt{\tau^2 + \tau^2 + \tau^2} = \sqrt{3}\,\tau = \frac{29404.3}{d^2}\left(1 + \frac{300}{d}\right)$$

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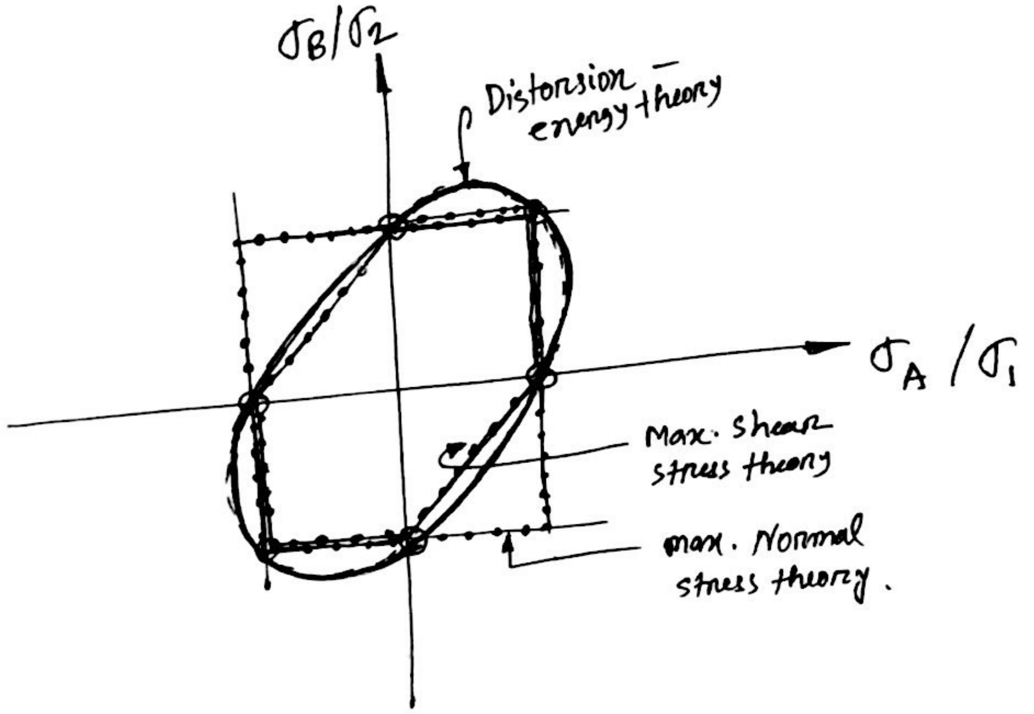
So, $N = \frac{\sigma_y}{\sigma} \Rightarrow \frac{29404.3}{d^2} + \frac{8.81 \times 10^6}{d^3} = \frac{400}{3}$

$\Rightarrow d = 42.29$ mm.

Conclusions:

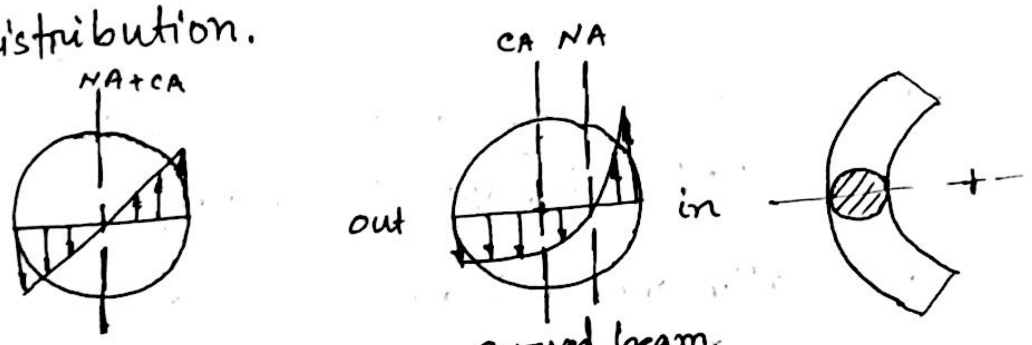
→ Maximum normal stress theory gives sometime unsafe value. So it is generally not used today.

→ Maximum shear stress theory gives good results that is comparable to distorsion energy theory. But shear stress theory gives always conservative value.



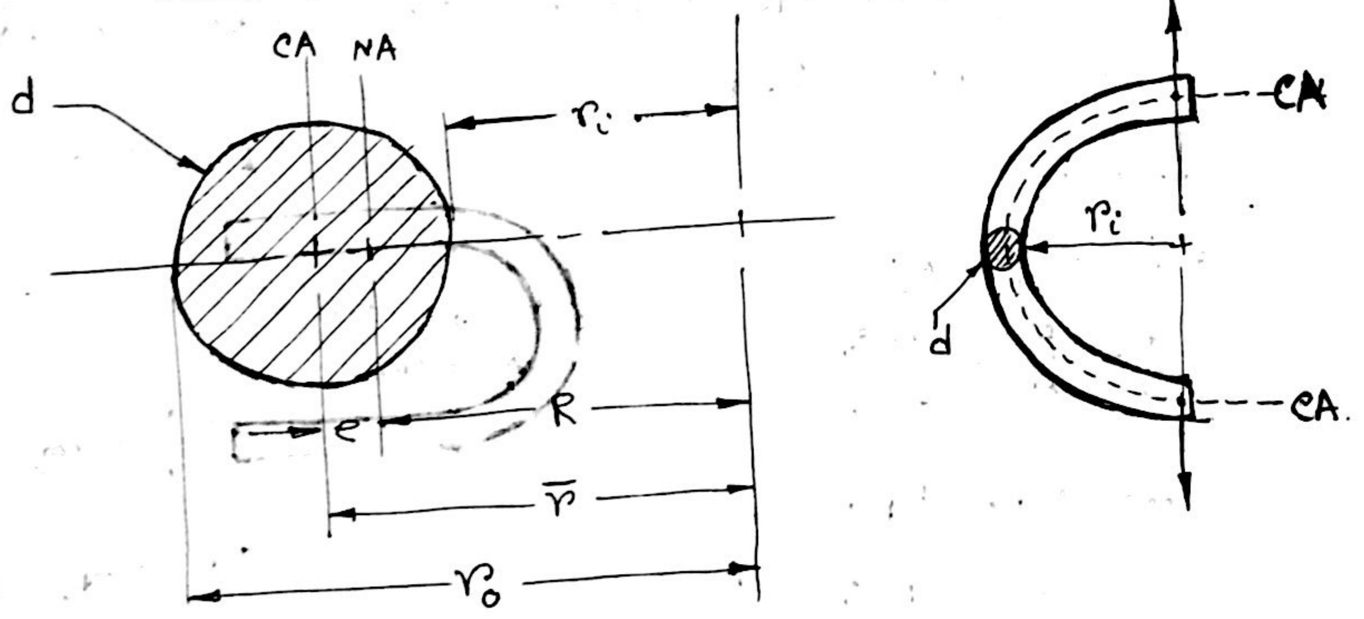
Curved Beam:

→ Initially curved, This causes non linear stress distribution.



Straight beam. curved beam.

→ The stress at inner fiber is maximum but the neutral axis moves toward inner fiber.



→ Here d, $\bar{r}$ are known from beam configuration. R is then calculated. R → depends on cross section.

$R = \dfrac{d^2}{4\left(2\bar{r} - \sqrt{4\bar{r}^2 - d^2}\right)}$	For circular section
$R = \dfrac{h}{\ln(r_o/r_i)}$	For rectangular section h = height of cross-section

→ Flexural stress due to bending:

$\sigma_i = \dfrac{Mc_i}{Aer_i}$	For inner fiber
$\sigma_o = \dfrac{Mc_o}{Aer_o}$	For outer fiber

→ M = Moment about centroidal axis.

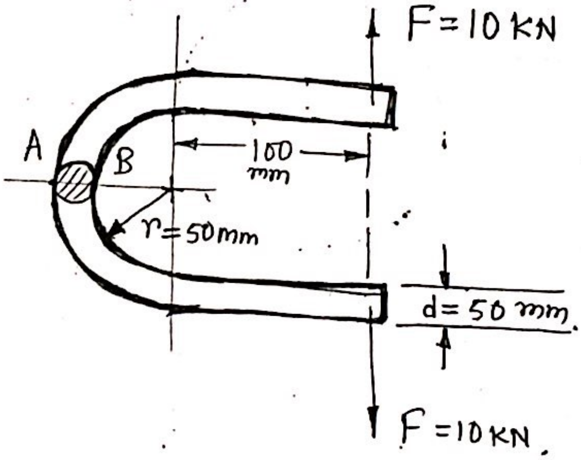
Not with respect to neutral axis.

→ Usually curved beams are always in combined stress.

→ c_i and c_o are distance of inner and outer fiber from neutral axis.

Problem: (47) Find the maximum stress at point A and B in the curved beam shown.

Solution:



Stress at point A:

$$\sigma_A = \frac{F}{A} - \frac{Mc_o}{Aer_o}$$

Stress at point B:

$$\sigma_B = \frac{F}{A} + \frac{Mc_i}{Aer_i}$$

Now, $R = \dfrac{d^2}{4(2\bar{r} - \sqrt{4\bar{r}^2 - d^2})}$

$$= \frac{50^2}{4(2\times75 - \sqrt{4\times75^2 - 50^2})} = 72.9 \text{ mm.}$$

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$e = \bar{r} - R$

$= 75 - 72.9 = 2.1$ mm.

$r_i = 50$ mm

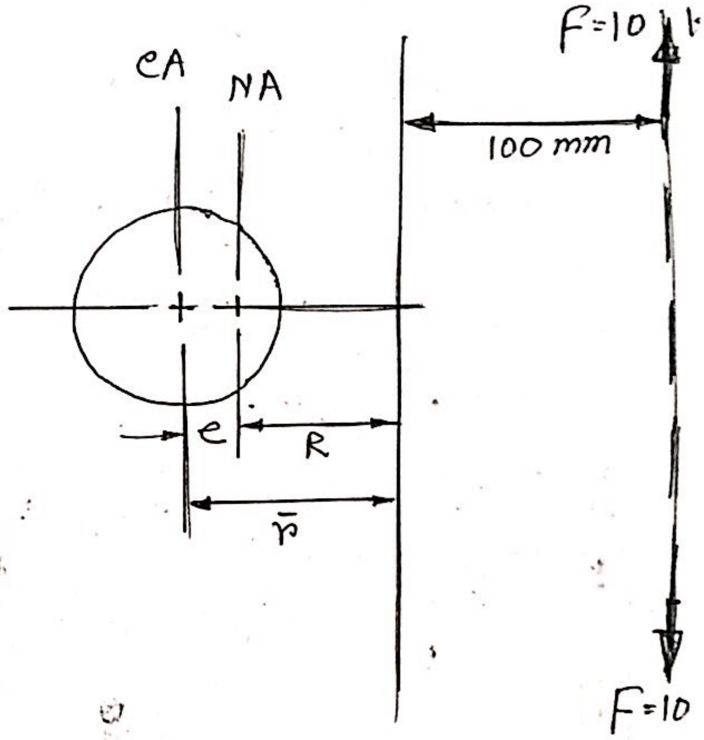
$r_o = 50 + 50 = 100$ mm.

$C_o = r_o - R$

$= 100 - 72.9 = 27.1$ mm

$C_i = R - r_i$

$= 72.9 - 50 = 22.9$ mm.



$A = \pi\left(\frac{d^2}{4}\right) = 198.9$ mm^2.

* $M = (10 \times 10^3) \times (100 + \bar{r})$ $\boxed{\text{About CA}}$

$= 10^4 \times (100 + 75) = 1.75 \times 10^6$ Nmm.

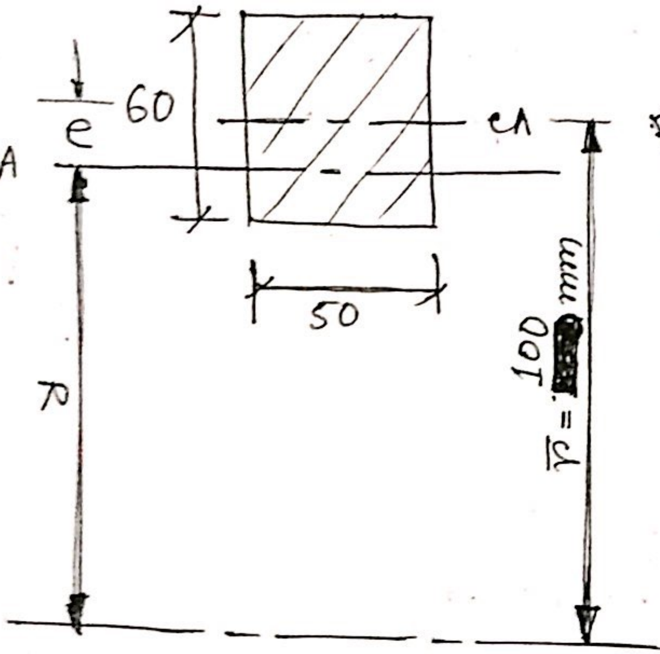
So, $\sigma_A = \frac{F}{A} - \left(\frac{MC_o}{Ae\,r_o}\right) = \frac{10^4}{198.9} - \left(\frac{1.75 \times 10^6 \times 27.1}{198.9 \times 2.1 \times 100}\right)$

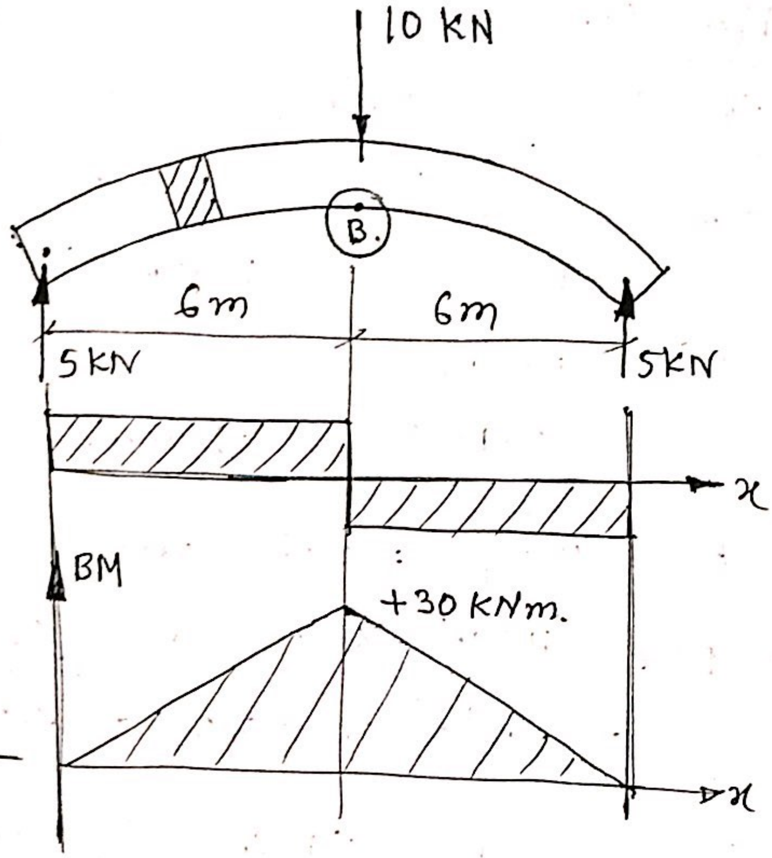
$= 50.3 - 1135.4 = -1085.1$ MPa.

$\sigma_B = \frac{F}{A} - \left(\frac{MC_i}{Ae\,r_i}\right) = \frac{10^4}{198.9} + \frac{1.75 \times 10^6 \times 22.9}{198.9 \times 2.1 \times 50}$

$= 50.3 + 1918.9 = 1969.2$ MPa.

Problem:(48) Find the factor safety for the beam shown in figure. $\sigma_y = 1400$ MPa. $\boxed{\text{To get idea}}$





Maximum stress will be developed at point B.

$$R = \frac{h}{\ln\left(\frac{r_o}{r_i}\right)} = \frac{50}{\ln\left(\frac{100+30}{100-30}\right)} = 80.6 \text{ mm.}$$

$$e = \bar{r} - R = (100 - 80.6) = 19.4 \text{ mm.}$$

$$r_i = \bar{r} - 30 = 70 \text{ mm.}$$

$$c_i = R - r_i = 80.6 - 70 = 10.6 \text{ mm.}$$

$$A = 60 \times 50 = 300 \text{ mm}^2$$

$$M = 30 \times 10^6 \text{ Nmm.}$$

$$\sigma = \frac{M c_i}{A e r_i} = \frac{30 \times 10^6 \times 10.6}{300 \times 19.4 \times 70} = 780 \text{ MPa.}$$

$$\therefore \text{ factor of safety, } N = \frac{\sigma_y}{\sigma} = \frac{1400}{780} = 1.8.$$

(Ans).

Pressure Vessel:

→ Confined box having pressure difference from outside to inside.

→ Based on thickness and size of the space classified in two catagories.

ⓐ Thin walled pressure vessel ($d > 20t$)

ⓑ Thick walled pressure vessel ($d < 20t$)

Thin walled pressure vessel: (cylindrical vessel)

→ Three components of stress are acted along three different direction.

σ_t → tangential stress / hoop stress / circumferential stress.

σ_l → longitudinal stress.

σ_r → Radial stress.

→ For thin walled pressure vessel σ_r is neglected.

$$\boxed{\sigma_t = \frac{PD}{2t}}$$

$$\boxed{\sigma_l = \frac{PD}{4t}}$$

P = inside pressure / pressure difference ($P_i - P_o$)

D = inner diameter

t = Thickness.

→ Always $\sigma_t > \sigma_l$; thats why σ_t is main reson to fail.

$$\boxed{\sigma_t = 2\sigma_l}$$

Problem: (49) Determine the bursting pressure for a pressure vessel that has diameter $d = 100$ mm and thickness $t = 2$ mm. The material strength is 500 MPa.

Solution: $\sigma_t = \frac{PD}{2t} = \frac{P \times 100}{2 \times 2} = 25P$

$$\sigma_l = \frac{PD}{4t} = \frac{P \times 100}{4 \times 2} = 12.5P.$$

So, to fail: $\sigma_t = \sigma_y$

$$\Rightarrow 25P = 500$$

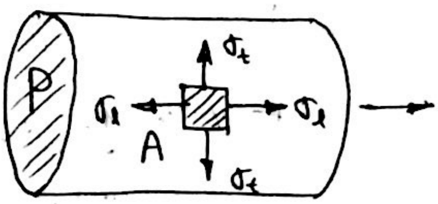
$$\Rightarrow P = 20 \ N/mm^2$$

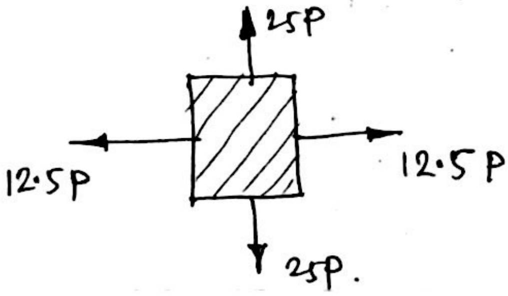
$$= 20 \times 10^6 \ N/m^2 \ (Pa)$$

$$= 197.4 \ (atm). \text{ Ans.}$$

Problem (50) Find the actual bursting pressure for problem 49. Considering combined effect and maximum shear stress theory. (Both 2D and 3D)

Solution: Consider an element A on the surface of the vessel.

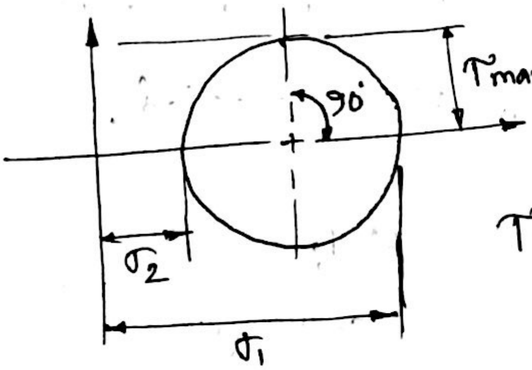




2D – analysis : Draw a mohr's circle with $\sigma_x = 12.5$ MPa and $\sigma_y = 25$ MPa. $\boxed{\text{Omiting } P}$

From figure :

$$\tau_{max} = 6.25\,(P)\ \text{MPa}$$



$\tau_{max} = 6.25$ MPa

Shear stress theory :

$$\tau_{max} = \frac{\tau_y}{2}$$

$\Rightarrow P = 40\ N/mm^2$ (surely Unsafe).

3D – analysis :

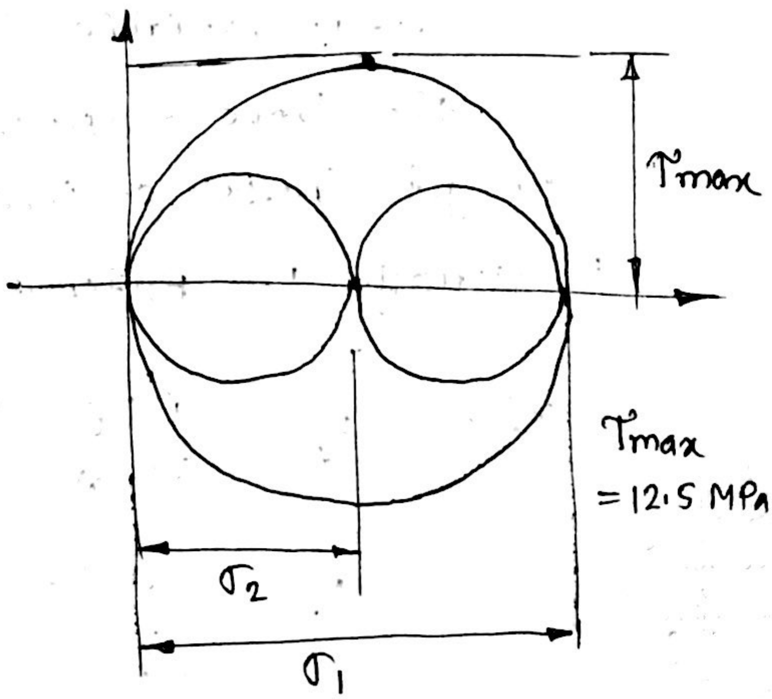
From figure :

$$\tau_{max} = 12.5\,(P)$$

Shear stress theory

$$\tau_{max} = \frac{\tau_y}{2}$$

$\Rightarrow P = 20\ N/mm^2$.



$\tau_{max} = 12.5$ MPa.

Thick walled pressure vessel (cylindrical)

- Thickness is comparable with dimensions. Thickness contribute to stress distribution
- σ_l is neglected.

$$\sigma_t = \frac{a^2 P_i - b^2 P_o}{(b^2 - a^2)} + \frac{a^2 b^2 (P_i - P_o)}{(b^2 - a^2) r^2}$$

$$\sigma_r = \frac{a^2 P_i - b^2 P_o}{(b^2 - a^2)} - \frac{a^2 b^2 (P_i - P_o)}{(b^2 - a^2) r^2}$$

Here, $P_i \rightarrow$ Internal pressure, $P_o \rightarrow$ outter pressure

$b \rightarrow$ external radius, $a \rightarrow$ internal radius

$r \rightarrow$ Variable along radial direction.

- In the above expressions put $r = a$ and b to get stresses at inner and outer surface.
- Usually P_o (External pressure) $= 0$ (gauge pressure)

Then, $\sigma_t = \frac{a^2 P_i}{(b^2 - a^2)} \left(1 + \frac{b^2}{r^2}\right)$

$$\sigma_r = \frac{a^2 P_i}{(b^2 - a^2)} \left(1 - \frac{b^2}{r^2}\right)$$

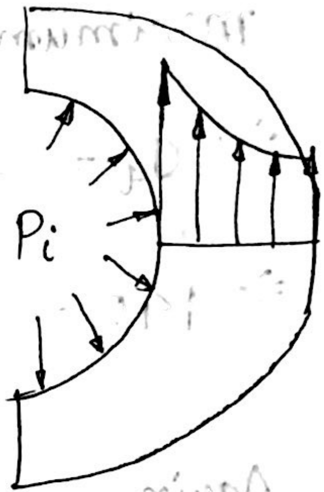
Distribution of σ_t and σ_r : $\boxed{P_o = 0 \text{ gauge pressure}}$

$$\sigma_t = \frac{a^2 P_i}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right)$$

at, $r = a$; $\sigma_t = \left(\frac{b^2 + a^2}{b^2 - a^2}\right) P_i$

at, $r = b$; $\sigma_t = \left(\frac{2a^2}{b^2 - a^2}\right) P_i$

$\boxed{\sigma_t \text{ nowhere is zero.}}$

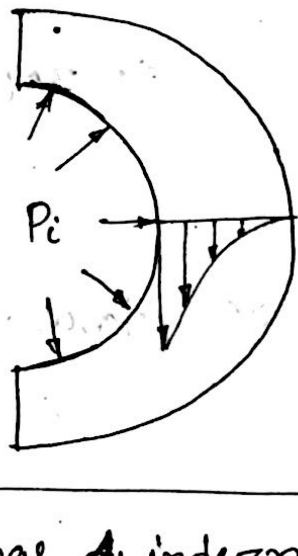


$$\sigma_r = \frac{a^2 P_i}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right)$$

at, $r = a$; $\sigma_r = -P_i$

at, $r = b$; $\sigma_r = 0$.

$\boxed{\sigma_r \text{ at outer surface is zero}}$



Problem: (51); A pressure vessel has internal diameter of 100 mm and thickness 20 mm. If the circumferential stress and radial stress are to be bellow 140 MPa and 300 MPa, then find the internal pressure.

Solution: ($\boxed{d/t = 5 > 20}$ so thick walled cylinder)

we know for internal pressure all stresses are maximum when $r = a$.

$$\therefore \sigma_t = \frac{a^2 P_i}{b^2 - a^2}\left(1 + \frac{b^2}{a^2}\right)$$

$$\Rightarrow 140 = \frac{100^2 P_i}{120^2 - 100^2}\left(1 + \frac{120^2}{100^2}\right)$$

$P_i = 25.25\ Nmm^{-2}$

Again, $$\sigma_r = \frac{a^2 P_i}{b^2 - a^2}\left(1 - \frac{b^2}{a^2}\right)$$

$$\Rightarrow 300 = \frac{100^2 P_i}{120^2 - 100^2}\left(1 - \frac{120^2}{100^2}\right)$$

$P_i = -300\ \frac{N}{mm^2}$

negetive sign does't mean vaccum

So, Maximum pressure it can withstand is $= 25.25 \left(\frac{N}{mm^2}\right) = 219.6$ atm

(Ans)

Reinforced Beam:

→ Some materials (like concrete, wood) are so weak in tension. The beam made by these materials are reinforced with ductile material like steel.

→ steel and concrete has a natural adhesion that enables no slip. And concrete has almost same coefficient of thermal expansion.

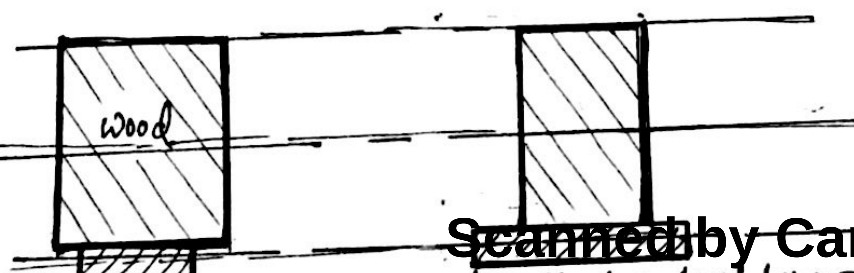
→ The former beam analysis is based on assumption of homogenous material. So those formulae are not applicable for reinforced beams.

→ But slight modification allows to use them. This modification includes replacing the steel section by an equivalent wood section or vice versa.

→ The area of the replaced section must be in a manner that it stands at same distance from the neutral axis.

→ The area of replaced section depends only on the modulus of elasticity of two materials.

consider a reinforced beam as shown in figure.



$$A_2 = \left(\frac{E_s}{E_w}\right) A_1$$

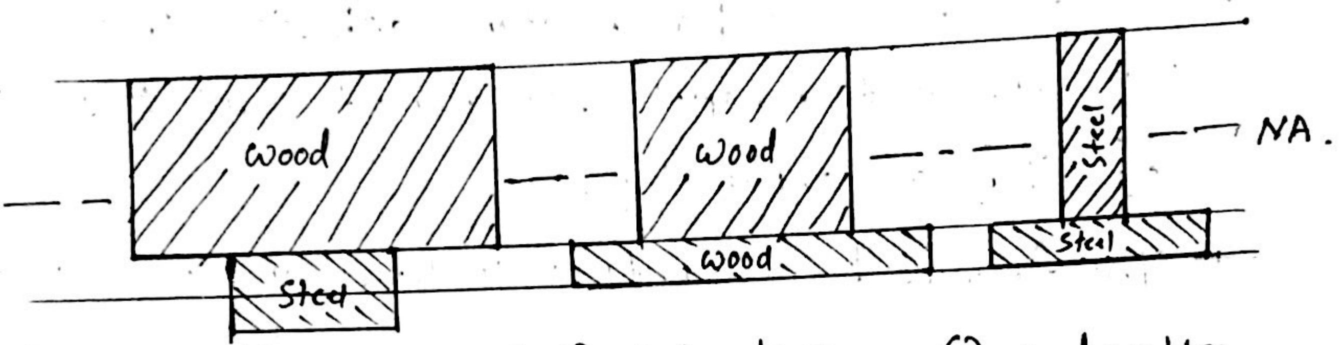
$$\Rightarrow A_2 = n A_1$$

A_2 - steel replaced by wood.

→ Two types of replacement are possible:

(a) steel section replaced by wood.

(b) wood section replaced by steel.



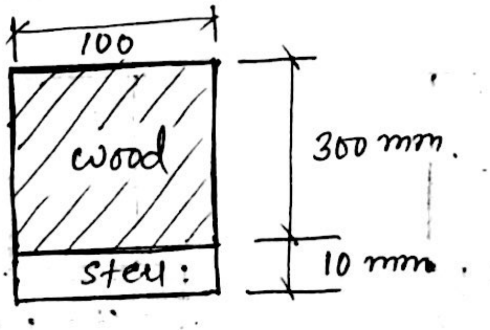
(a) Original reinforced beam

(b) steel section replaced by equivalent wood.

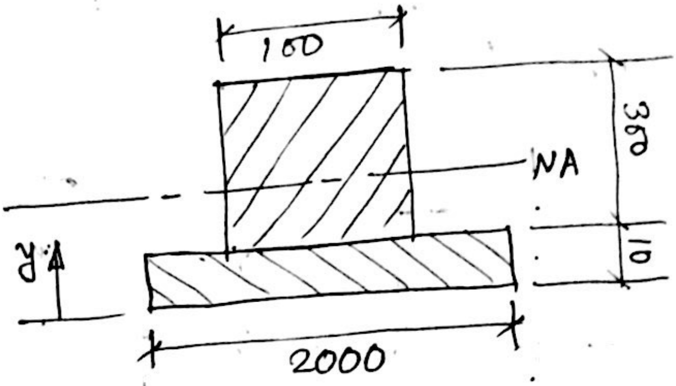
(c) wood section replaced by equivalent steel section.

→ In either way cant change the height of any section. Because they must be at same distance from Neutral axis.

Problem: (52) Determine the maximum resisting moment applicable to the beam shown. The stress in wood and steel must be kept lower than or equal to 10 MPa and 150 MPa. [$n=20$]



Solution: The wood equivalent section is:



$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A}$$

$$= \frac{(20000 \times 5) + (30000 \times 160)}{(20000 + 30000)} = 98 \text{ mm.}$$

$$I = \frac{1}{12}(20000 \times 10^3) + 20000 \times 93^2 + \frac{1}{12}(100 \times 300^3)$$

$$+ 30000 \times 62^2$$

$$= 3.0 \times 10^9 \text{ mm}^4$$

Based on wood stress:

$$M = \left(\frac{\sigma I}{c}\right)_{wood}$$

$$= \left(\frac{60 \times 3 \times 10^9}{212}\right)$$

$$= 141 \text{ KNm.}$$

Based on steel stress:

$$M = \left(\frac{\sigma I}{c}\right)_{steel}$$

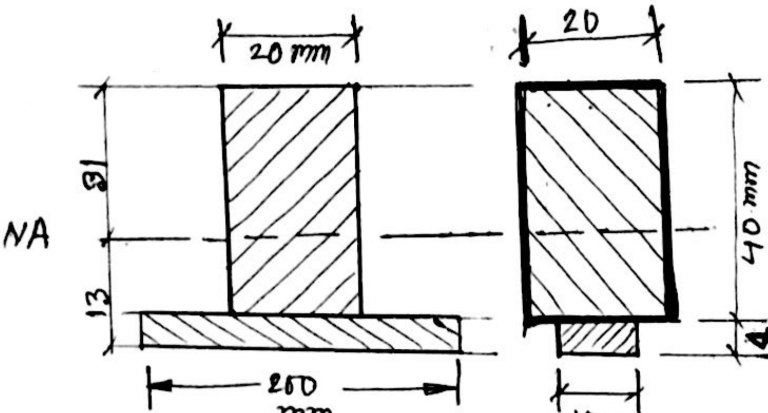
$$= \left(\frac{\sigma_s I_w}{n\, c_s}\right) = \frac{150 \times 3 \times 10^9}{20 \times 98}$$

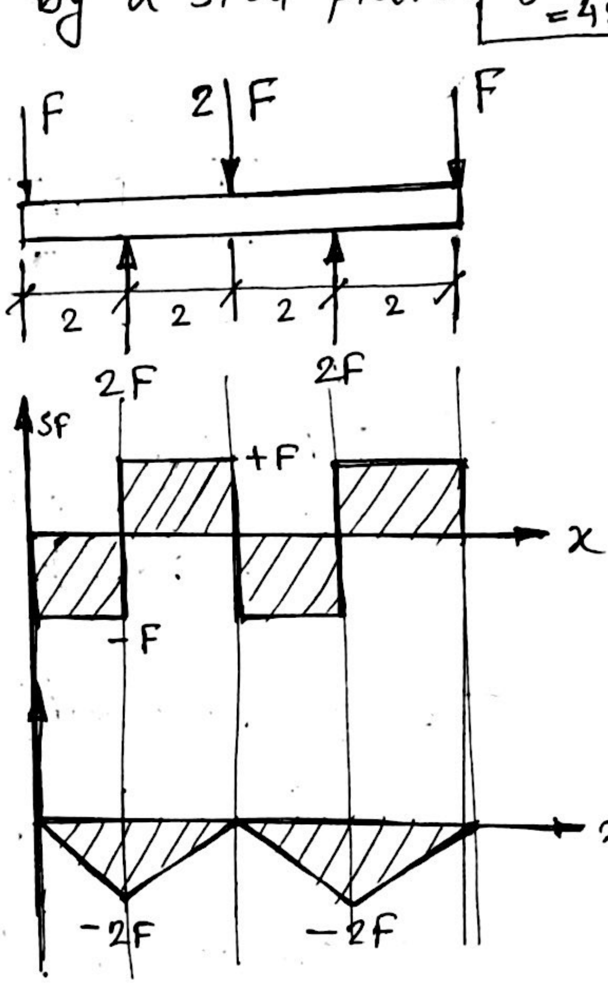
$$= 229.6 \text{ KNm.}$$

So, Maximum safe moment = 141 KNm.

(Access steel is used as reinforcement)

Problem: (53) For the beam shown below find maximum value of Force F. The beam is rainforced as shown in figure $[n = 20]$ by a steel plate. $\boxed{\sigma_s = 15\sigma_w = 450\,\text{MPa}}$





$$\bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A}$$

$$= \left(\frac{800\times 2 + 800\times 24}{800+800}\right)$$

$$= 13 \text{ mm.}$$

$$I = \frac{1}{12}(200\times 4^3) + 800\times(11)^2$$

$$+ \frac{1}{12}(20\times 40^3) + 800\times(11)^2$$

$$= 301.3\times 10^3 \text{ mm}^4.$$

$$M_{max} = 2F \text{ Nm.}$$

$$= 2F\times 10^3 \text{ Nmm.}$$

considering wood strength:

$$2F = \left(\frac{\sigma I}{c}\right)_{wood}; \Rightarrow 2F = \left\{\frac{30\times 301.3\times 10^3}{31}\right\}\times 10^{-3}$$

$$\Rightarrow F = 145.8 \text{ N}$$

Considering steel strength:

$$2F = \left(\frac{\sigma I}{c}\right)_{steel} = \frac{\sigma_s I_w}{n c_s}$$

$$\Rightarrow 2F = \left\{\frac{450 \times 301.3 \times 10^3}{20 \times 13}\right\} \times 10^{-3}$$

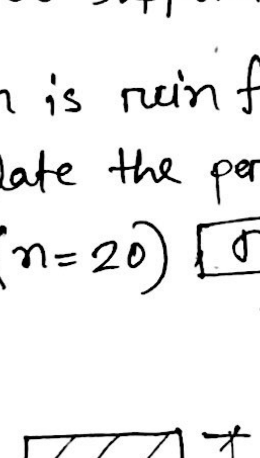
$$\Rightarrow F = 260.7 \text{ N.}$$

$\therefore$ Maximum allowable force $F = 145.8$ KN

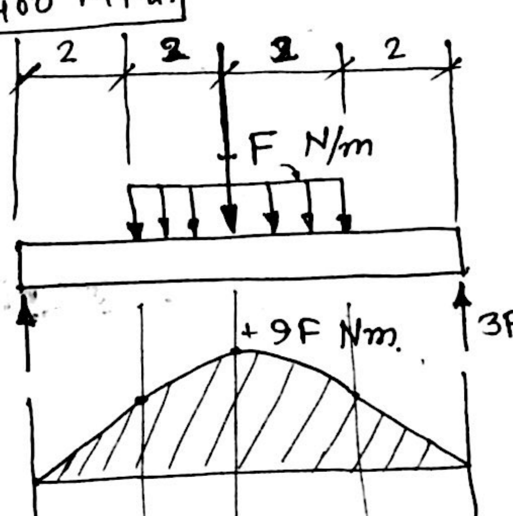
<u>Problem: (54)</u> A beam is made of wood ($\sigma_y = 20$ MPa) and loaded as shown. Find the maximum value of F that can be supported by the beam.

If the beam is reinforced with steel as shown then calculate the percent increase in load capability. ($n = 20$) $\boxed{\sigma_s = 400 \text{ MPa.}}$

$$I = \frac{1}{12} 10 \times 20^3$$
$$= 6.67 \times 10^3 \text{ mm}^4.$$



Non reinforced section.



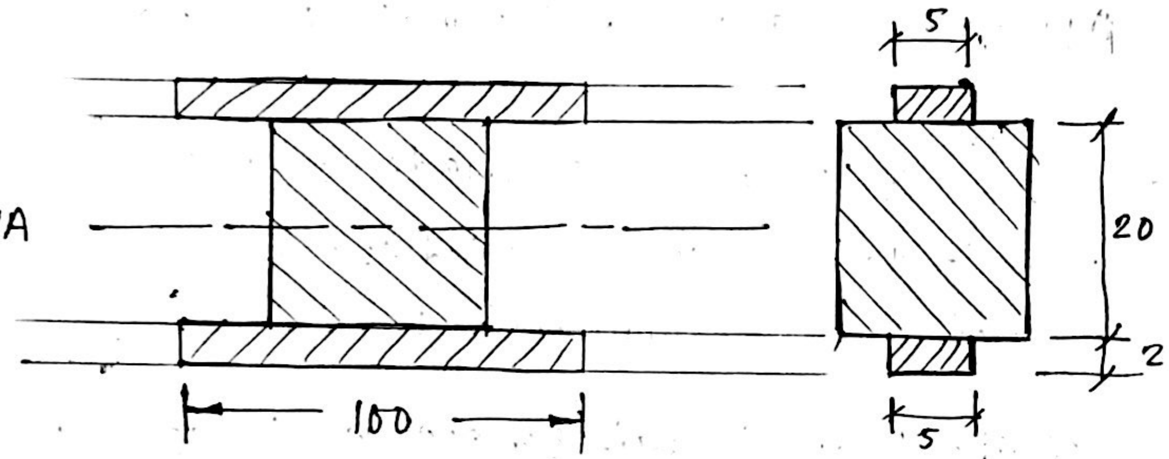
With out reinforcement:

$$M = \frac{\sigma I}{c}$$

$$\Rightarrow \quad 9F \times 10^3 = \frac{20 \times 6.67 \times 10^3}{10}$$

$$\Rightarrow \quad F = 1.48 \text{ N.}$$

With reinforcement:



Equivalent wood section.

reinforced section.

$$I = \left(\frac{1}{12} \times 10 \times 20^3\right) + 2\left[\frac{1}{12}(100) \times 2^3 + 200 \times 11^2\right]$$

$$= 55200 \text{ mm}^4$$

Based on wood strength: $M = \left(\frac{\sigma I}{c}\right)_{wood}$

$$\Rightarrow \quad 9 \times 10^3 F = \left(\frac{20 \times 55200}{10}\right)$$

$$\Rightarrow \quad F = 12.26 \text{ N.}$$

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Based on steel strength: $M = \left(\frac{\sigma I}{c}\right)_s = \left(\frac{\sigma_s I_w}{n c_s}\right)$

$$\Rightarrow 9\times10^3\ F = \left(\frac{400\times55200}{20\times12}\right) \Rightarrow F = 10.22\ N.$$

So, maximum allowable force $= 10.22\ N$.

$\boxed{\text{Under reinforced}}$

% increase in load capability $= \left(\frac{10.22-1.48}{1.48}\right)\times100\%$

$= 590\ \%$.

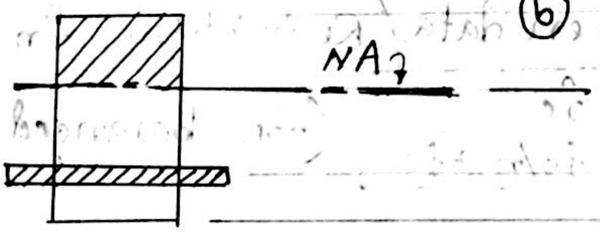
* **considering no-slip: (Reinforced concrete Beam)**

→ These beams are called RCC beam.

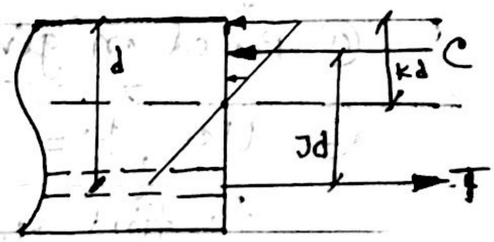
→ concrete can't withstand tensile stress. So it actually holds the reinforcement in the tensile side of beam.

→ The effective areas are shown in figure ⓐ

ⓐ



ⓑ



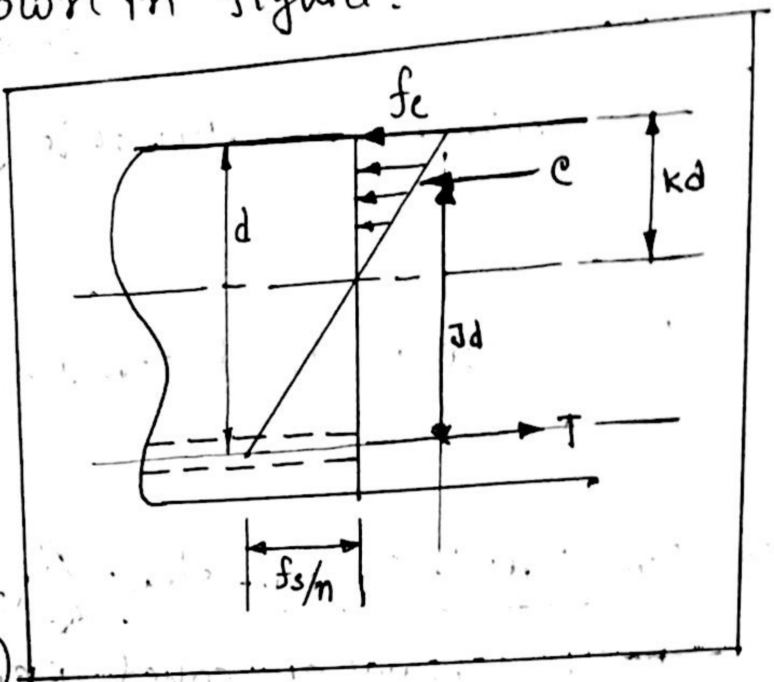
→ let, A_s (steel area) is (d) mm beneath the top surface. Neutral axis is (kd) mm beneath the top surface and the moment arm (Jd) mm.

$$\boxed{Jd = d - \frac{1}{3}kd}$$

→ To design in balance moment way the stress on top fiber must be equal to strength of concrete (f_c)

and the stress in steel must be equal to strength of steel (f_s). shown in figure.

→ From the figure:



$$\frac{kd}{d} = \frac{f_c}{f_s/n + f_c}$$

$$\Rightarrow \boxed{k = \frac{f_c}{f_s/n + f_c}}$$

→ At least one of parameter (b or d) must be known or assumed. Assumptions are based on some restrictions to ~~f~~ physical dimensions.

→ Design procedure:

ⓐ Based on given data / Restrictions find "b" or "d".

ⓑ Use $\boxed{k = \frac{f_c}{f_s/n + f_c}}$ (for balanced design) to find k.

ⓒ After knowing k, use $\boxed{j = \left(1 - \frac{1}{3}k\right)}$ to find 'j'.

ⓓ Then use moment equation: (To find 'b' or 'd'.)

$$\boxed{M = C(jd) = \left(\tfrac{1}{2}f_c\right)(kd \cdot b)(jd)}$$

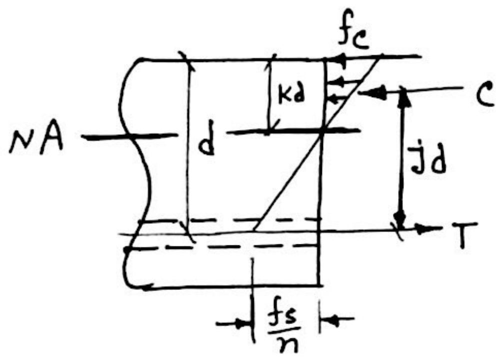
ⓔ Then use Force equation to find A_s^e.

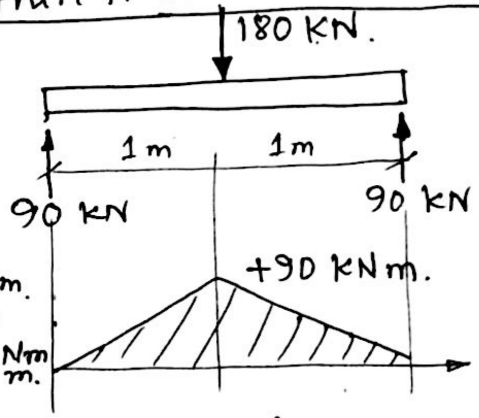
$$\boxed{A_s^e \cdot f_s = \tfrac{1}{2} \cdot f_c \, (kd)\, b}$$

Problem: (55) Design the RCC beam for folowing loading condition. The width of the beam must not exceed 250 mm. The strength of concrete of the beam is 12 MPa while the reinforcing steel rod has a strength of 140 MPa. [redacted] The modulus of elasticity are $E_c = 20$ GPa, $E_s = 200$ GPa.

$\boxed{\text{If the diameter of steel rod} = 5\text{ mm then find no of rod reinforce}}$

Solution: $n = \frac{200}{20} = 10.$





$M = 90$ KNm.

$= 90 \times 10^6$ Nmm.

Here, $f_c = 12$ MPa. $f_s = 140$ MPa.

step ①: From figure, $\frac{kd}{d} = \frac{f_c}{f_s/n + f_c}$; $k = 0.545.$

②: So, $Jd = d - \frac{1}{3}kd.$; $J = 0.818.$

③: Moment equation: $M = \frac{1}{2} f_c \cdot (kd) b \cdot (jd)$

$\Rightarrow 90 \times 10^6 = 2.675\ bd^2$; $bd^2 = 33.65 \times 1$

④: Assume $b = 220$ mm < 250 mm.

$\therefore d = 391$ mm.

⑤: Force equation: $A_s^e \cdot f_s = \frac{1}{2} f_c \cdot kd \cdot b$

$\Rightarrow A_s^e = 2010\ mm^2$

$\therefore A_s = (A_s^e / n) = 201\ mm^2.$

So, let assume no of steel rods = η_s.

then, $\eta_s \left(\frac{\pi d_{st.rod}^2}{4}\right) = A_s$

$$\Rightarrow \eta_s = \left\{\frac{201 \times 4}{\pi \times 5^2}\right\} = 10.2$$

$$\approx 11. \text{ (Ans)}$$

So, the designed beam is: